Convolution semigroups on compact groups - transition kernels, traces and asymptotics.

Dave Applebaum (Sheffield)

Let \((\mu_t, t \geq 0)\) be a weakly continuous convolution semigroup of probability measures on a compact group \(G\). Let \((T_t, t \geq 0)\) be the associated Feller-Markov semigroup of linear operators on the \(L^2\)-space of normalised Haar measure. We show that \(T_t\) is trace-class for \(t > 0\) if and only if \(\mu_t\) has an \(L^2\)-density \(k_t\). If \(\mu_t\) is central then the co-ordinate functions of irreducible representations are a complete orthogonal set of eigenfunctions for \(T_t\) and if \(k_t\) is also continuous we have

\[
k_t(e) = \text{tr}(T_t) = \sum_{\pi} d_{\pi}^2 \lambda_{\pi}(t),
\]

where \(d_{\pi}\) is the dimension of the irreducible representation \(\pi\) and \(\lambda_{\pi}(t)\) is the corresponding eigenvalue of \(T_t\). We compute the small-time asymptotics of \(k_t(e)\) on \(SU(2)\) when \(\mu_t\) has a Cauchy distribution.