

# Bornhuetter–Ferguson as a General Principle of Loss Reserving

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## Run-Off Triangles of Cumulative Losses (1)

An example from the *Claims Reserving Manual*:

Accident Year	Development Year					
	0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					

The enumeration of the development years represents delays with respect to the accident years.

## Run-Off Triangles of Cumulative Losses (2)

Accident Year	Development Year								
	0	1	...	$k$	...	$n-i$	...	$n-1$	$n$
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$i$	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
$n$	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss  $S_{i,k}$  is said to be

- ▶ **observable** if  $i + k \leq n$ .
- ▶ **non-observable** or **future** if  $i + k > n$ .
- ▶ **current** if  $i + k = n$ .
- ▶ **ultimate** if  $k = n$ .

## Run-Off Triangles of Cumulative Losses (3)

The purpose of loss reserving is to **predict**

- ▶ the ultimate losses  $S_{i,n}$  and
- ▶ the accident year reserves  $S_{i,n} - S_{i,n-i}$

More generally: The aim is to predict

- ▶ the future cumulative losses  $S_{i,k}$
- ▶ the future incremental losses  $Z_{i,k} := S_{i,k} - S_{i,k-1}$
- ▶ the calendar year reserves  $\sum_{j=p-n}^n Z_{j,p-j}$
- ▶ the total reserve  $\sum_{j=1}^n \sum_{l=n-j+1}^n Z_{j,l}$

with  $i + k \geq n + 1$  and  $p = n + 1, \dots, 2n$ .

Thus: **The principal task** considered here is to predict the future cumulative losses.

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# Development Patterns

- ▶ A **development pattern for quotas** consists of parameters  $\gamma_0, \gamma_1, \dots, \gamma_n$  with

$$\gamma_k = E[S_{i,k}]/E[S_{i,n}]$$

for all  $k = 0, 1, \dots, n$  and for all  $i = 0, 1, \dots, n$ .

These parameters are called **development quotas** (percentages reported).

- ▶ A **development pattern for factors** consists of parameters  $\varphi_1, \dots, \varphi_n$  with

$$\varphi_k = E[S_{i,k}]/E[S_{i,k-1}]$$

for all  $k = 1, \dots, n$  and for all  $i = 0, 1, \dots, n$ .

These parameters are called **development factors** (age-to-age factors).

# Development Patterns: Cumulative Losses and Quotas

Accident Year	Development Year					
	0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4044
2	1265	2433	3233	3977	4477	4677
3	1490	2873	3880	4880	5380	5680
4	1725	3261	4361	5461	5961	6361
5	1889	3489	4889	5889	6489	6889
0	0.287	0.533	0.696	0.858	0.958	1.000
1	0.275	0.520	0.686	0.846	0.951	1.000
2	0.270	0.520	0.691	0.850	0.957	1.000
3	0.262	0.506	0.683	0.859	0.947	1.000
4	0.271	0.513	0.686	0.859	0.937	1.000
5	0.274	0.506	0.710	0.855	0.942	1.000

# Development Patterns: Cumulative Losses and Factors

Accident Year	Development Year					
	0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4044
2	1265	2433	3233	3977	4477	4677
3	1490	2873	3880	4880	5380	5680
4	1725	3261	4361	5461	5961	6361
5	1889	3489	4889	5889	6489	6889
0		1.853	1.306	1.233	1.116	1.044
1		1.889	1.319	1.234	1.123	1.052
2		1.923	1.329	1.230	1.126	1.045
3		1.928	1.351	1.258	1.102	1.056
4		1.890	1.337	1.252	1.092	1.067
5		1.847	1.401	1.205	1.102	1.062

## Development Patterns: Quotas and Factors

- ▶ If the parameters  $\gamma_0, \gamma_1, \dots, \gamma_n$  form a development pattern for quotas, then the parameters  $\varphi_1, \dots, \varphi_n$  with

$$\varphi_k := \frac{\gamma_k}{\gamma_{k-1}}$$

form a development pattern for factors.

- ▶ If the parameters  $\varphi_1, \dots, \varphi_n$  form a development pattern for factors, then the parameters  $\gamma_0, \gamma_1, \dots, \gamma_n$  with

$$\gamma_k := \prod_{l=k+1}^n \frac{1}{\varphi_l}$$

form a development pattern for quotas.

## Development Patterns: Estimation of Quotas

For estimation of the parameter  $\gamma_k$  of a development pattern for quotas, the only obvious estimator provided by the run-off triangle is the **empirical individual quota**

$$\hat{\gamma}_{0,k} := S_{0,k}/S_{0,n}$$

Accident Year	Development Year					
	0	1	2	3	4	5
0	0.287	0.533	0.696	0.858	0.958	1.000
1	0.275	0.520	0.686	0.846	0.951	1.000
2	0.270	0.520	0.691	0.850	0.957	1.000
3	0.262	0.506	0.683	0.859	0.947	1.000
4	0.271	0.513	0.686	0.859	0.937	1.000
5	0.274	0.506	0.710	0.855	0.942	1.000

## Development Patterns: Estimation of Factors

For estimation of the parameter  $\varphi_k$  of a development pattern for factors, the run-off triangle provides the **empirical individual factors**

$$\hat{\varphi}_{i,k} := S_{i,k}/S_{i,k-1}$$

with  $i = 0, 1, \dots, n - k$ . Moreover, any weighted mean of these estimators is an estimator as well.

Accident Year	Development Year					
	0	1	2	3	4	5
0		1.853	1.306	1.233	1.116	1.044
1		1.889	1.319	1.234	1.123	1.052
2		1.923	1.329	1.230	1.126	1.045
3		1.928	1.351	1.258	1.102	1.056
4		1.890	1.337	1.252	1.092	1.067
5		1.847	1.401	1.205	1.102	1.062

# Development Patterns: Chain–Ladder Factors

The chain–ladder factors

$$\hat{\varphi}_k^{\text{CL}} := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}} = \sum_{j=0}^{n-k} \frac{S_{j,k-1}}{\sum_{h=0}^{n-k} S_{h,k-1}} \hat{\varphi}_{j,k}$$

are weighted means and may be used to estimate the development factors  $\varphi_k$ .

Accident Year $i$	Development Year $k$					
	0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					
$\hat{\varphi}_k^{\text{CL}}$		1.899	1.329	1.232	1.120	1.044

# Development Patterns: Chain–Ladder Quotas

The chain–ladder quotas

$$\hat{\gamma}_k^{\text{CL}} := \prod_{l=k+1}^n \frac{1}{\hat{\varphi}_l^{\text{CL}}}$$

may be used to estimate the development quotas  $\gamma_k$ .

Accident Year $i$	Development Year $k$					
	0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					
$\hat{\varphi}_k^{\text{CL}}$		1.899	1.329	1.232	1.120	1.044
$\hat{\gamma}_k^{\text{CL}}$	0.278	0.527	0.701	0.864	0.968	1

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# The original Bornhuetter–Ferguson Method (1)

In its original version, the Bornhuetter–Ferguson method aims at predicting **calendar year reserves**

$$R_i := S_{i,n} - S_{i,n-i}$$

If  $\gamma_0, \gamma_1, \dots, \gamma_n$  is a **development pattern for quotas**, then the expected reserves satisfy the model equation

$$E[R_i] = (1 - \gamma_{n-i}) E[S_{i,n}]$$

The **original Bornhuetter–Ferguson predictors** of the reserves  $R_i$  are defined as

$$\hat{R}_i := (1 - \hat{\gamma}_{n-i}^{\text{CL}}) \pi_i \hat{\kappa}_i$$

where

- ▶  $\hat{\gamma}_{n-i}^{\text{CL}}$  is the current chain–ladder quota,
- ▶  $\pi_i$  is a volume measure, and
- ▶  $\hat{\kappa}_i$  is an estimator of the **expected loss ratio**  $\kappa_i := E[S_{i,n}/\pi_i]$

## The original Bornhuetter–Ferguson Method (2)

- ▶ Transformation into predictors of the ultimate losses  $S_{i,n}$ :

$$\widehat{S}_{i,n} := S_{i,n-i} + \left(1 - \widehat{\gamma}_{n-i}^{\text{CL}}\right) \pi_j \widehat{K}_j$$

- ▶ Transformation into predictors of other future cumulative losses  $S_{i,k}$ :

$$\widehat{S}_{i,k} := S_{i,n-i} + \left(\widehat{\gamma}_k^{\text{CL}} - \widehat{\gamma}_{n-i}^{\text{CL}}\right) \pi_j \widehat{K}_j$$

Idea:

- ▶ Replace the chain–ladder quotas by arbitrary estimators of the quotas.
- ▶ Replace the estimators  $\pi_j \widehat{K}_j$  by arbitrary estimators of the expected ultimate losses  $E[S_{i,n}]$ .

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# The extended Bornhuetter–Ferguson Method (1)

The extended Bornhuetter–Ferguson method is based on the assumption, that there exists a **development pattern for quotas** and that

- ▶ **prior estimators**

$$\hat{\gamma}_0, \hat{\gamma}_1, \dots, \hat{\gamma}_n$$

(with  $\hat{\gamma}_n = 1$ ) of the development quotas  $\gamma_0, \gamma_1, \dots, \gamma_n$  and

- ▶ **prior estimators**

$$\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_n$$

of the **expected ultimate losses**

$$\alpha_i := E[S_{i,n}]$$

with  $i = 0, 1, \dots, n$

are available.

## The extended Bornhuetter–Ferguson Method (2)

These prior estimators can be obtained from

- ▶ **internal information** (provided by the run–off triangle, like chain–ladder factors),
- ▶ **volume measures** (like premiums) for the portfolio under consideration,
- ▶ **external information** (market statistics or data from similar portfolios) or
- ▶ a combination of these data.

## The extended Bornhuetter–Ferguson Method (3)

The future cumulative losses satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] + (\gamma_k - \gamma_{n-i})E[S_{i,n}]$$

Accordingly, the **extended Bornhuetter–Ferguson predictors** of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\text{BF}} := S_{i,n-i} + (\widehat{\gamma}_k - \widehat{\gamma}_{n-i})\widehat{\alpha}_i$$

Thus:

- ▶ The run-off triangle provides information perhaps only via the current losses.
- ▶ The predictors of the ultimate losses are obtained by linear extrapolation from the current losses.

# The extended Bornhuetter–Ferguson Method (4)

Accident Year $i$	Development Year $k$						$\hat{\alpha}_i$
	0	1	2	3	4	5	
0						3483	3517
1					3844	4043	3981
2				3977	4391	4621	4598
3			3880	4785	4389	5577	5658
4		3261	4442	5436	5995	6306	6214
5	1889	3344	4546	5558	6127	6443	6325
$\hat{\gamma}_k$	0.280	0.510	0.700	0.860	0.950	1.000	
$1 - \hat{\gamma}_k$	0.720	0.490	0.300	0.140	0.050	0.000	

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# The Loss–Development Method (1)

The loss–development method is based on the assumption, that there exists a **development pattern for quotas** and that **prior estimators**

$$\hat{\gamma}_0, \hat{\gamma}_1, \dots, \hat{\gamma}_n$$

(with  $\hat{\gamma}_n = 1$ ) of the development quotas  $\gamma_0, \gamma_1, \dots, \gamma_n$  are available.

The loss–development method does **not** involve any prior estimators for the expected ultimate losses.

## The Loss–Development Method (2)

The future cumulative losses satisfy the model equation

$$E[S_{i,k}] = \gamma_k \frac{E[S_{i,n-i}]}{\gamma_{n-i}}$$

Accordingly, the **loss–development predictors** of the future cumulative losses are defined as

$$\hat{S}_{i,k}^{\text{LD}} := \hat{\gamma}_k \frac{S_{i,n-i}}{\hat{\gamma}_{n-i}}$$

Thus:

- ▶ The run–off triangle provides information perhaps only via the current losses.
- ▶ The predictors of the ultimate losses are obtained by scaling the current losses.
- ▶ The predictors of other future cumulative losses are obtained by scaling the predictors of the ultimate losses.

# The Loss–Development Method (3)

Accident Year $i$	Development Year $k$					
	0	1	2	3	4	5
0						3483
1					3844	4046
2				3977	4393	4624
3			3880	4767	5266	5543
4		3261	4476	5499	6074	6394
5	1889	3440	4722	5802	6409	6746
$\hat{\gamma}_k$	0,280	0,510	0,700	0,860	0,950	1,000

## The Loss–Development Method (4)

Because of the definition

$$\widehat{S}_{i,k}^{\text{LD}} := \widehat{\gamma}_k \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

the loss–development predictors can be written as

$$\widehat{S}_{i,k}^{\text{LD}} = S_{i,n-i} + \left( \widehat{\gamma}_k - \widehat{\gamma}_{n-i} \right) \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

In this form, the loss–development predictors attain the shape of the [extended Bornhuetter–Ferguson predictors](#) with respect to the prior estimators

$$\widehat{\alpha}_i^{\text{LD}} := \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

of the expected ultimate losses.

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# The Chain–Ladder Method (1)

The chain–ladder method is based on the assumption that there exists a **development pattern for factors**.

The chain–ladder method relies **completely** on the observable cumulative losses of the run–off triangle and involves **no** prior estimators at all.

As estimators of the development factors, the chain–ladder method uses the chain–ladder factors

$$\hat{\varphi}_k^{\text{CL}} := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}} = \sum_{j=0}^{n-k} \frac{S_{j,k-1}}{\sum_{h=0}^{n-k} S_{h,k-1}} \hat{\varphi}_{j,k}$$

## The Chain–Ladder Method (2)

The future cumulative losses  $S_{i,k}$  satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] \prod_{l=n-i+1}^k \varphi_l$$

Accordingly, the **chain–ladder predictors** of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\text{CL}} := S_{i,n-i} \prod_{l=n-i+1}^k \widehat{\varphi}_l^{\text{CL}}$$

Thus:

- ▶ The chain–ladder method consists in successive scaling of the current loss  $S_{i,n-i}$  to the level of the future cumulative loss  $S_{i,k}$ .

# The Chain–Ladder Method (3)

Accident Year $i$	Development Year $k$					
	0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4013
2	1265	2433	3233	3977	4454	4650
3	1490	2873	3880	4780	5354	5590
4	1725	3261	4334	5339	5980	6243
5	1889	3587	4767	5873	6578	6867
$\hat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

## The Chain–Ladder Method (4)

Because of the definition

$$\widehat{S}_{i,k}^{\text{CL}} := S_{i,n-i} \prod_{l=n-i+1}^k \widehat{\varphi}_l^{\text{CL}}$$

the chain–ladder predictors of the future cumulative losses can be written as

$$\widehat{S}_{i,k}^{\text{CL}} = \widehat{\gamma}_k^{\text{CL}} \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

In this form, the chain–ladder predictors attain the shape of the **loss–development predictors** with respect to the **chain–ladder quotas**.

## The Chain–Ladder Method (5)

Since

$$\widehat{S}_{i,k}^{\text{CL}} = \widehat{\gamma}_k^{\text{CL}} \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

the chain–ladder predictors of the future cumulative losses can also be written as

$$\widehat{S}_{i,k}^{\text{CL}} = S_{i,n-i} + \left( \widehat{\gamma}_k^{\text{CL}} - \widehat{\gamma}_{n-i}^{\text{CL}} \right) \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

In this form, the chain–ladder predictors attain the shape of the **extended Bornhuetter–Ferguson predictors** with respect to the **chain–ladder quotas** and the prior estimators

$$\widehat{\alpha}_i^{\text{CL}} := \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

of the expected ultimate losses.

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# The Cape Cod Method (1)

The Cape Cod method is based on the assumption, that there exists a **development pattern for quotas** and that **prior estimators**

$$\widehat{\gamma}_0, \widehat{\gamma}_1, \dots, \widehat{\gamma}_n$$

(with  $\widehat{\gamma}_n = 1$ ) of the development quotas  $\gamma_0, \gamma_1, \dots, \gamma_n$  are available.

It is also based on the assumption that there exist **volume measures**  $\pi_0, \pi_1, \dots, \pi_n$  for the accident years and that the expected ultimate loss ratio

$$\kappa := E \left[ \frac{S_{i,n}}{\pi_i} \right]$$

is the same for all accident years.

## The Cape Cod Method (2)

The future cumulative losses satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] + (\gamma_k - \gamma_{n-i})\pi_i \kappa$$

Accordingly, the **Cape Cod predictors** of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\text{CC}} := S_{i,n-i} + (\widehat{\gamma}_k - \widehat{\gamma}_{n-i})\pi_i \widehat{\kappa}^{\text{CC}}(\boldsymbol{\pi}, \widehat{\boldsymbol{\gamma}})$$

where

$$\widehat{\kappa}^{\text{CC}}(\boldsymbol{\pi}, \widehat{\boldsymbol{\gamma}}) := \frac{\sum_{j=0}^n S_{j,n-j}}{\sum_{j=0}^n \pi_j \widehat{\gamma}_{n-j}}$$

is the **Cape Cod loss ratio**.

## The Cape Cod Method (3)

Therefore, the Cape Cod predictors of the future cumulative losses have the shape of the **extended Bornhuetter–Ferguson predictors** with respect to the Cape Cod estimators

$$\hat{\alpha}_i^{\text{CC}} := \pi_i \hat{\kappa}_i^{\text{CC}}(\boldsymbol{\pi}, \hat{\boldsymbol{\gamma}})$$

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## The Additive Method (1)

The additive method (or **incremental loss ratio method**) is based on the assumption, that there exist

- ▶ **volume measures**  $\pi_0, \pi_1, \dots, \pi_n$  for the accident years, and
- ▶ parameters  $\zeta_0, \zeta_1, \dots, \zeta_n$  such that the **expected incremental loss ratio**

$$\zeta_k := E \left[ \frac{Z_{i,k}}{\pi_i} \right]$$

is the same for all accident years, where

$$Z_{i,k} := \begin{cases} S_{i,0} & \text{if } k = 0 \\ S_{i,k} - S_{i,k-1} & \text{else} \end{cases}$$

is the **incremental loss** of accident year  $i$  and development year  $k$ .

## The Additive Method (2)

The cumulative and incremental losses satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] + \pi_i \sum_{l=n-i+1}^k \zeta_l$$

Accordingly, the **additive predictors** of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\text{AD}} := S_{i,n-i} + \pi_i \sum_{l=n-i+1}^k \widehat{\zeta}_l^{\text{AD}}$$

where

$$\widehat{\zeta}_k^{\text{AD}} := \frac{\sum_{j=0}^{n-k} Z_{j,k}}{\sum_{j=0}^{n-k} \pi_j}$$

is the **additive incremental loss ratio** of development year  $k$ .

## The Additive Method (3)

Since

$$\widehat{S}_{i,k}^{\text{AD}} := S_{i,n-i} + \pi_i \sum_{l=n-i+1}^k \widehat{\zeta}_l^{\text{AD}}$$

the additive predictors can be written as

$$\widehat{S}_{i,k}^{\text{AD}} := S_{i,n-i} + \left( \frac{\sum_{l=0}^k \widehat{\zeta}_l^{\text{AD}}}{\sum_{l=0}^n \widehat{\zeta}_l^{\text{AD}}} - \frac{\sum_{l=0}^{n-i} \widehat{\zeta}_l^{\text{AD}}}{\sum_{l=0}^n \widehat{\zeta}_l^{\text{AD}}} \right) \left( \pi_i \sum_{l=0}^n \widehat{\zeta}_l^{\text{AD}} \right)$$

or as

$$\widehat{S}_{i,k}^{\text{AD}} := S_{i,n-i} + \left( \widehat{\gamma}_k^{\text{AD}}(\boldsymbol{\pi}) - \widehat{\gamma}_{n-i}^{\text{AD}}(\boldsymbol{\pi}) \right) \widehat{\alpha}_i^{\text{AD}}(\boldsymbol{\pi})$$

In this form, the additive predictors of the future cumulative losses have the shape of the **extended Bornhuetter–Ferguson predictors** with respect to the **additive quotas**  $\widehat{\gamma}_k^{\text{AD}}(\boldsymbol{\pi})$  and the additive estimators  $\widehat{\alpha}_i^{\text{AD}}(\boldsymbol{\pi})$  of the expected ultimate losses.

# The Additive Method (4)

Remark:

It can be shown that

$$\hat{\alpha}_i^{\text{AD}}(\boldsymbol{\pi}) = \hat{\alpha}_i^{\text{CC}}(\boldsymbol{\pi}, \hat{\gamma}^{\text{AD}}(\boldsymbol{\pi}))$$

such that the additive method can be viewed as the Cape Cod method with respect to the volume measures  $\boldsymbol{\pi}$  and the additive quotas  $\hat{\gamma}^{\text{AD}}(\boldsymbol{\pi})$ .

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# The Bornhuetter–Ferguson Principle (1)

Comparison of certain versions of the extended Bornhuetter–Ferguson method:

Prior Estimators of Expected Ultimate Losses	Prior Estimators of Cumulative Quotas		
	$\hat{\gamma}^{\text{external}}$	$\hat{\gamma}^{\text{CL}}$	$\hat{\gamma}^{\text{AD}}(\pi)$
$\hat{\alpha}^{\text{external}}$	Bornhuetter– Ferguson Method (external)		
$\hat{\alpha}^{\text{LD}}(\hat{\gamma})$	Loss–Development Method (external)	Chain–Ladder Method	
$\hat{\alpha}^{\text{CC}}(\pi, \hat{\gamma})$	Cape Cod Method (external)		Additive Method

## The Bornhuetter–Ferguson Principle (2)

The **Bornhuetter–Ferguson principle** consists of

- ▶ an **analytic** part,  
in which known methods of loss reserving are interpreted as **versions** of the extended Bornhuetter–Ferguson method,
- ▶ a **synthetic** part,  
in which components of different versions of the extended Bornhuetter–Ferguson method are used to construct **new versions** of the extended Bornhuetter–Ferguson method, and
- ▶ the **simultaneous application** of several versions of the extended Bornhuetter–Ferguson method to a given run–off triangle of cumulative losses.

## The Bornhuetter–Ferguson Principle (3)

Application of the **Bornhuetter–Ferguson principle** may result in

- ▶ the selection of **reliable predictors**,
- ▶ the selection of **reliable ranges**,
- ▶ the comparison of the given portfolio with a **market portfolio**, and
- ▶ the control of **pricing**.

In either case, careful actuarial judgement of the quality of the sources of information underlying the different versions of the extended Bornhuetter–Ferguson method is essential.

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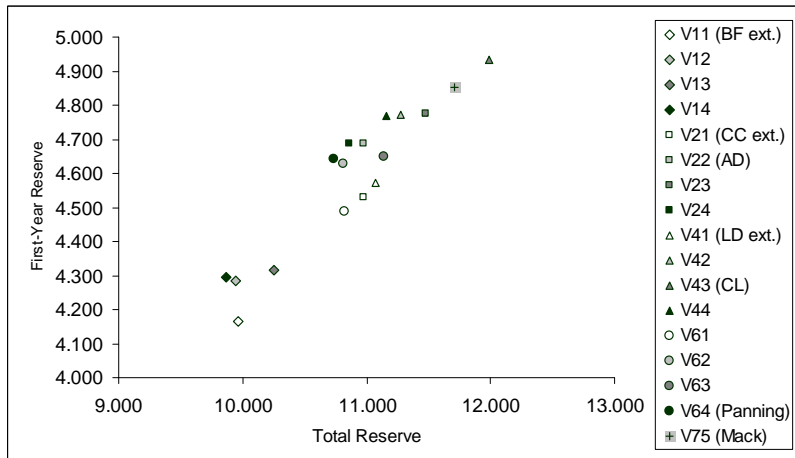
# An Example

Modified example:

Acc. Year $i$	Development Year $k$						$\pi_i$	$\hat{\alpha}_i^{\text{ext}}$
	0	1	2	3	4	5		
0	1001	1855	2423	2988	3335	3483	4000	3520
1	1113	2103	2774	3422	3844		4500	3980
2	1265	2433	3233	3977			5300	4620
3	1490	2873	3880				6000	5660
4	1725	4261					6900	6210
5	1889						8200	6330
$\hat{\gamma}_k^{\text{ext}}$	0.2800	0.5300	0.7100	0.8600	0.9500	1.0000		

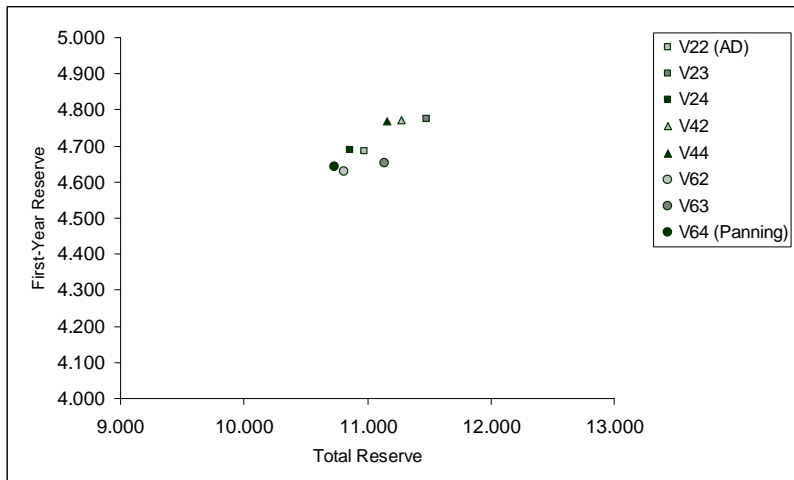
# An Example

Predictors of various versions:



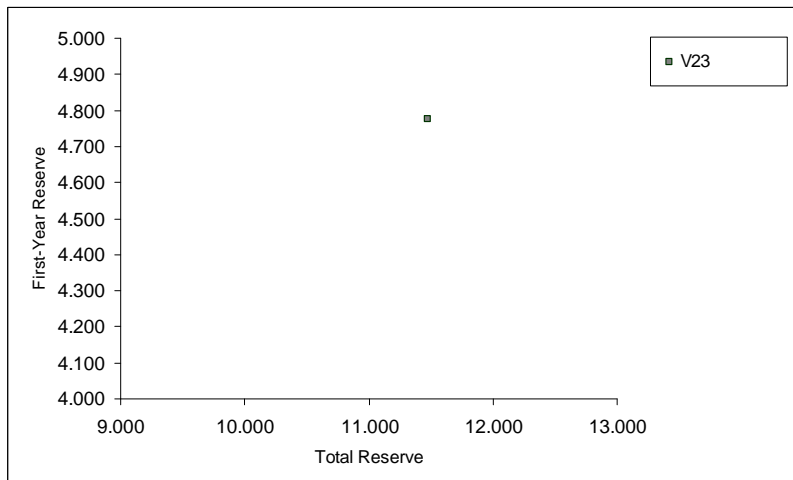
# An Example

Reliable predictors:



# An Example

Selected predictor:



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