The stationary tail index of contractive iterated function systems

Gerold Alsmeyer
Universität Münster

Abstract

Let $(X_n)_{n\geq 0}$ be a contractive iterated function system (IFS) on a complete separable metric space $(\mathbb{X}, d)$ with unbounded metric d, i.e.

$$X_n = \Psi_n \circ ... \circ \Psi_1(X_0)$$

for $n \geq 1$, where $\Psi_1, \Psi_2, ...$ are iid random Lipschitz functions on $\mathbb{X}$ with Lipschitz constants $L(\Psi_1), L(\Psi_2), ...$ Let π denote the unique stationary distribution of $(X_n)_{n\geq 0}$ and $x_0 \in \mathbb{X}$ an arbitrary reference point. Assuming $\mathbb{P}_\pi(d(x_0, X_0) > r) > 0$ for all $r > 0$, we will provide bounds for the lower and upper tail index ϑ_* and ϑ^* of $d(x_0, X_0)$ in equilibrium (under $\mathbb{P}_\pi$), defined by

$$\vartheta_* := -\limsup_{x\to\infty} \frac{\log \mathbb{P}(X > x)}{\log x} \quad \text{and} \quad \vartheta^* := -\liminf_{x\to\infty} \frac{\log \mathbb{P}(X > x)}{\log x}.$$

This will be done by providing lower and upper bounds for $d(x_0, X_n)$ under $\mathbb{P}_\pi$ in terms of rather simple IFS on $\mathbb{R}_\geq$ and the use of Goldie's implicit renewal theorem [2]. Special attention is paid to the particularly relevant case when $\mathbb{X} = \mathbb{R}$. The method is illustrated by some examples including the well-known AR(1) model with ARCH(1) errors which has been studied earlier in some detail by Borkovec and Klüppelberg [1].

References

[1] Borkovec, M., Klüppelberg, C.: The tail of the stationary distribution of an autoregressive process with ARCH(1) errors. Ann. Appl. Probab. **11**(4), 1220–1241 (2001)

[2] Goldie, C.M.: Implicit renewal theory and tails of solutions of random equations. Ann. Appl. Probab. **1**(1), 126–166 (1991)