zu.gast@math

TU Dresden
Fachrichtung Mathematik

Prof. Dr. Christian Berg
University of Copenhagen

Vortragsreihe

Indeterminate moment problems and the theory of entire functions (Part I – V)

16.10.2014 (Do)	13:00-14:30 Uhr, WIL A 124	Part I (AG Stochastik & Analysis)
17.10.2014 (Fr)	13:00-14:30 Uhr, WIL C 203	PART II (Graduate Lectures)
23.10.2014 (Do)	13:00-14:30 Uhr, WIL A 124	Part III (AG Stochastik & Analysis)
24.10.2104 (Fr)	13:00-14:30 Uhr, WIL C 203	PART IV (Graduate Lectures)
30.10.2014 (Do)	13:00-14:30 Uhr, WIL A 124	Part V (AG Stochastik & Analysis)

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Indeterminate moment problems and the theory of entire functions

Christian Berg

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Abstract

A positive measure μ on the real line with moments of any order

$$s_n = \int x^n \, d\mu(x), n = 0, 1, \ldots,$$

can be indeterminate in the sense that there exist positive measures $\nu \neq \mu$ with the same moments. This was discovered by Stieltjes.

Examples are not obvious, but here is an important one: Given $0 < q < 1$ the log-normal moments are $s_n = q^{-n(n+2)/2}$ given by

$$\frac{\sqrt{q}}{\sqrt{2\pi \log(1/q)}} \int_0^\infty x^n \exp\left(-\frac{(\log x)^2}{2\log(1/q)} \right) dx.$$

Defining

$$h(x) = \sin\left(\frac{2\pi}{\log(1/q)} \log x\right)$$

then the non-negative densities $(-1 \leq r \leq 1)$

$$\frac{\sqrt{q}}{\sqrt{2\pi \log(1/q)}} \exp\left(-\frac{(\log x)^2}{2\log(1/q)} \right) [1 + rh(x)]$$

and the discrete measures $a > 0$

$$\frac{1}{L(a)} \sum_{k=-\infty}^{\infty} a^k q^{k(k+2)/2} \delta_{aq^k}$$

all have the log-normal moments.

One can describe all the solutions to an indeterminate moment problem by the so-called Nevanlinna parametrization. This is based on four entire functions A, B, C, D defined in terms of the orthonormal polynomials $P_n, n \geq 0$ associated with the measure. The parameter space is a class of holomorphic functions in the upper half-plane called Pick functions.

The polynomials P_n are characterized by the three-term recurrence relation

$$xP_n(x) = b_n P_{n+1}(x) + a_n P_n(x) + b_{n-1} P_{n-1}(x), \quad n \geq 0, \tag{1}$$

for certain sequences $a_n \in \mathbb{R}, b_n > 0, n \geq 0$.

In the lectures I will discuss this as well as the theory of order of growth of entire holomorphic functions. The goal is to give an exposition of recent results from [3], which describes the growth of the entire functions A, B, C, D associated with an indeterminate moment problem in terms of the recurrence coefficients $(a_n), (b_n)$ from (1) or the moments.

Basic information can be found in [1], [2] and [4].

References

[1] N. I. Akhiezer, *The Classical Moment Problem and Some Related Questions in Analysis*. English translation, Oliver and Boyd, Edinburgh, 1965.

[2] C. Berg, *Indeterminate moment problems and the theory of entire functions*, J. Comput. Appl. Math. **65** (1995), 27–55.

[3] C. Berg and R. Szwarc, *On the order of indeterminate moment problems*, Advances in Mathematics **250** (2014), 105–143.

[4] B. Ya. Levin, *Lectures on entire functions*, American Mathematical Society, Providence, R.I., 1996.

University of Copenhagen, Denmark
berg@math.ku.dk