START 2014
Workshop on Stochastic Analysis And Related Topics

Schedule and Abstracts

22nd - 23rd September 2014
Room: WIL C 207
Willersbau, Zellescher Weg 12-14
01069 Dresden
## Schedule:

**Monday, September 22nd**

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<td>Krzysztof Bogdan</td>
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<td></td>
<td>(Wroclaw)</td>
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<td>10.00-10.45</td>
<td>Nikola Sandric</td>
<td>Long-time behavior of Lévy-type processes: recurrence, transience and ergodicity</td>
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<td>11.15-12.00</td>
<td>Peter Scheffler</td>
<td>Governing equations of infinite mean Lévy Walks</td>
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<td><strong>Lunch</strong></td>
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<td>14.00-14.30</td>
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<td>Alexander Steinicke</td>
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<td>Stefan Geiß</td>
<td>Decoupling on the Wiener space and applications to BSDEs</td>
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<td>16.15-17.00</td>
<td>Eulalia Nualart</td>
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<td>A regularity result for quasilinear parabolic SPDEs</td>
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The dinner takes place at the **Restaurant Budapest** in Hofmühlenstr. 14, 01187 Dresden. We go there together (by bus) and meet at 18.30 in front of the lecture room C207.
### Tuesday, September 23rd

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<td>Eva Löcherbach</td>
<td>What makes neurons spike- the stochastic Hodgkin-Huxley model</td>
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<td>14.00-14.45</td>
<td>Tadeusz Kulczycki</td>
<td>On concavity of the expected value of the first exit time of the Cauchy process</td>
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<td>14.45-15.30</td>
<td>Serge Cohen</td>
<td>What happens to a random walk of chained particles when the chain is very long?</td>
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<td>16.00-16.30</td>
<td>Nil Venet</td>
<td>There is no fractional Brownian fields indexed by the cylinder</td>
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<td>16.30-17.15</td>
<td>Jan Kallsen</td>
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Multiple Lévy systems are related to multiple Mecke-Palm formulas. I will explain the connection and if time permits we will also discuss moment formulas for stochastic integrals and applications to Fourier multipliers.

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**Long-time behavior of Lévy-type processes: recurrence, transience and ergodicity**

**Nikola Sandrić, University of Zagreb & TU Dresden**

An $\mathbb{R}^d$-valued Lévy process $(\{L_t\}_{t \geq 0}, \{\mathbb{P}^x\}_{x \in \mathbb{R}^d})$ is said to be recurrent if
\[
\mathbb{E}^0 \left[ \int_0^\infty 1_{\{|L_t|<a\}} dt \right] = \infty \quad \text{for all } a > 0
\]
and transient if
\[
\mathbb{E}^0 \left[ \int_0^\infty 1_{\{|L_t|<a\}} dt \right] < \infty \quad \text{for all } a > 0.
\]

It is well known that every Lévy process is either recurrent or transient. If $q(\xi)$ is the characteristic exponent of $\{L_t\}_{t \geq 0}$, then the recurrence and transience property of $\{L_t\}_{t \geq 0}$ is characterized by the Chung-Fuchs criterion, that is, $\{L_t\}_{t \geq 0}$ is recurrent if, and only if,
\[
\int_{\{\xi:|\xi|<r\}} \text{Re} \left( \frac{1}{q(\xi)} \right) d\xi = \infty \quad \text{for some } r > 0.
\]

In this talk, we present Chung-Fuchs type conditions for the recurrence and transience of Feller processes associated with pseudo-differential operators. Let $(\{F_t\}_{t \geq 0}, \{\mathbb{P}^x\}_{x \in \mathbb{R}^d})$ be an $\mathbb{R}^d$-valued Feller process. It is well known that if the test functions $C^\infty_c(\mathbb{R}^d)$ are contained in the domain of the generator of $\{F_t\}_{t \geq 0}$, then the generator is a pseudo-differential operator and a so-called symbol $q(x, \xi)$ is associated with this operator. We show that if $\{F_t\}_{t \geq 0}$ is a Feller process with symbol $q(x, \xi)$, then, under an irreducibility condition and some additional mild technical conditions, $\{F_t\}_{t \geq 0}$ is recurrent if
\[
\int_{\{\xi:|\xi|<r\}} \frac{d\xi}{\sup_{x \in \mathbb{R}^d}|q(x, \xi)|} = \infty \quad \text{for some } r > 0.
\]

Chung-Fuchs type condition for the transience of Feller processes which admits a symbol and satisfy
\[
\mathbb{E}^x \left[ e^{i(F_t-x,\xi)} \right] = \text{Re} \mathbb{E}^x \left[ e^{i(F_t-x,\xi)} \right], \quad t \geq 0, \quad x, \xi \in \mathbb{R}^d,
\]
\[
\int_{\{\xi:|\xi|<r\}} \frac{d\xi}{\inf_{x \in \mathbb{R}^d} q(x, \xi)} < \infty \quad \text{for some } r > 0.
\]
Further, by using the Chung-Fuchs type conditions, we discuss the recurrence and transience of Feller processes with respect to the dimension of the state space and Pruitt indices and the recurrence and transience of Feller-Dynkin diffusions and stable-like process (in the sense of R. Bass). In the one-dimensional symmetric case, we discuss perturbations of Feller processes which do not affect their recurrence and transience properties, and we present sufficient conditions for their recurrence and transience in terms of the corresponding Lévy measure. In addition, we present some comparison conditions for recurrence and transience also in terms of the Lévy measures. At the end, we discuss the (strong) ergodicity property of Feller processes.

Governing equations of infinite mean Lévy walks

Peter Scheffler, University of Siegen

The Lévy Walk is the process with continuous sample paths which arises from consecutive linear motions of i.i.d. lengths with i.i.d. directions. Assuming speed 1 and motions in the domain of $\beta$-stable attraction, we prove functional limit theorems and derive governing pseudo-differential equations for the law of the walker’s position. Both Lévy Walk and its limit process are continuous and ballistic in the case $\beta \in (0, 1)$.

Gaussian estimates for Schrödinger perturbations

Karol Szczypkowski, Wrocław University of Technology

A perturbation series is an explicit method of constructing new semigroups or fundamental solutions. It is thus of the interest to obtain its upper and lower bounds.

We propose a new general method of estimating Schrödinger perturbations of transition densities using an auxiliary transition density as a majorant of the perturbation series. We present applications to Gaussian bounds by proving an optimal 4G Theorem for the Gaussian kernel, the inequality which is a non-trivial extension of the so called 3G or 3P Theorem (as well known, 3P fails in its primary form for the Gaussian kernel). Further applications concern transition density of $1/2$ stable subordinator.
Aspects of Malliavin differentiation of random functions in the Lévy setting

Alexander Steinicke, University of Innsbruck

Consider the setting of a Lévy process $X = (X_t)_{t \in [0,T]}$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We study properties and applications of the Malliavin derivative defined by help of Itô’s chaos decomposition. The following question arises, for example, in the study of (backward) stochastic differential equations: Let

$$H : \Omega \times \mathbb{R}^d \to \mathbb{R}$$

be jointly measurable, for any $y \in \mathbb{R}^d$ we assume $H(\cdot, y)$ to be a random element of the Malliavin Sobolev space $D_{1,2}$, and for all $\omega \in \Omega$ let $H(\omega, \cdot) \in C^1(\mathbb{R}^d)$. If $G_1, \ldots, G_d \in D_{1,2}$, under which assumption do we get

$$H(\cdot, G_1, \ldots, G_d) \in D_{1,2}?$$

We answer this question and show how far chain rules may be extended to random variables of type $H(\cdot, G_1, \ldots, G_d)$. The results are achieved by investigating separately the Brownian and the jump direction of the Malliavin derivative.

Decoupling on the Wiener space and applications to BSDEs

Stefan Geiss, University of Innsbruck

We consider a decoupling technique on the Wiener space to define anisotropic Besov spaces, i.e. Banach spaces that describe the fractional smoothness of a random variable in certain 'directions'. The class of spaces, we obtain, contains the classical Besov spaces generated by real interpolation, but also new spaces that are needed to investigate the $L^p$-variation of the solution $(Y, Z)$ of a Backward Stochastic Differential Equation (BSDE)

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)ds - \int_t^T Z_s dW_s,$$

where the generator $f$ is of Lipschitz or quadratic type, and the BSDE is driven by a $d$-dimensional Brownian motion $W$. The point is, that there are no structural assumptions on the terminal condition $\xi$ imposed, so that $\xi$ might be a path-dependent functional of a forward diffusion.

By measuring the anisotropic singularities of $\xi$ and $f$ we obtain assertions about the $L^p$-variation of the solution $(Y, Z)$. As an intermediate step in proving our results we derive that our anisotropic Besov spaces are stable with respect to non-linear expectations generated by BSDEs.

The talk is based on

Noise excitability of the stochastic heat equation

Eulalia Nualart, Barcelona GSE

We study the behaviour of the moments and the Lyapunov exponent of the stochastic heat equation with Dirichlet boundary conditions depending on the amount of noise. Joint work with M. Foondun.

A regularity result for quasilinear parabolic SPDE’s

Martina Hofmanova, MPI Leipzig

We consider a quasilinear parabolic stochastic partial differential equation driven by a multiplicative noise and study regularity properties of its weak solution satisfying classical a priori estimates. In particular, we determine conditions on coefficients and initial data under which the weak solution is Hölder continuous in time and possesses spatial regularity that is only limited by the regularity of the given data. Our proof is based on an efficient method of increasing regularity: the solution is rewritten as the sum of two processes, one solves a linear parabolic SPDE with the same noise term as the original model problem whereas the other solves a linear parabolic PDE with random coefficients. This way, the required regularity can be achieved by repeatedly making use of known techniques for stochastic convolutions and deterministic PDEs. It is a joint work with Arnaud Debussche and Sylvain de Moor.

Prediction of Lévy-driven CARMA processes

Peter Brockwell, Colorado State University

In this talk we consider the problem of determining the conditional expectations, $E(Y(h)|Y(u), -\infty < u \leq 0)$ and $E(Y(h)|Y(u), -M \leq u \leq 0)$ where $h > 0$, $0 < M < \infty$ and $(Y(t))_{t \in \mathbb{R}}$ is a continuous-time ARMA (CARMA) process driven by a Lévy process $L$ with $E[L(1)] < \infty$. If the driving Lévy process satisfies $E(L(1)^2) < \infty$ then these are the minimum mean-squared error predictors of $Y(h)$ given $(Y(t))_{t \leq 0}$ and $(Y(t))_{-M \leq t \leq 0}$ respectively. In the course of the derivations we establish conditions under which the sample-path of $L$ can be recovered from that of $Y$, both when $Y$ is causal and strictly stationary and, without these assumptions, when $L$ is a pure-jump Lévy process. When $E(L(1)^2) < \infty$ and $Y$ is causal and strictly stationary we also determine the best linear predictors $P(Y(h)|Y(u), u \leq 0)$ and $P(Y(h)|Y(-n\Delta), n \in \mathbb{N})$, comparing their performance with that of $E(Y(h)|Y(u), -\infty < u \leq 0)$. Finally we use the expression for $P(Y(h)|Y(-n\Delta), n \in \mathbb{N})$ to establish a very simple algorithm for determining the parameters of the ARMA process obtained by sampling the CARMA process at regular intervals.
In this talk we give a survey about some common methods to show the absolute continuity of the probability law of the solution to SDEs and SPDEs. The most-used approach to this problem is to show that the random variables that form the solution are Malliavin differentiable and then apply an absolute continuity criterion such as the Bouleau-Hirsch criterion, see [4] for an overview. However, in recent years other approaches have appeared that also work under less restrictive hypotheses. We will present the following three approaches to show that a random variable $X$ is absolutely continuous:

1. the approach in [3] that uses the auxiliary random variable $G := \langle DX, -DL^{-1}X \rangle_H$, where $D$ is the Malliavin derivative operator and $L$ is Ornstein-Uhlenbeck operator,

2. the approach in [2] that uses the characteristic function of $X$, and

3. the approach in [1], where one uses Besov-space methods to show the existence of a density.

To each of these approaches we will also show, using the stochastic wave equation, how it can be applied in concrete settings.

References:


What makes neurons spike - the stochastic Hodgkin-Huxley model

Eva Löcherbach, University of Cergy

This is a joint work with Michèle Thieullen (Paris) and Reinhard Höpfner (Mainz).

The deterministic Hodgkin-Huxley model for the membrane potential of a single neuron describes the mechanism of spike generation (spikes = emission of action potentials) in response to an external input. We study a stochastic version of this model in which a cortical neuron receives some T-periodic (unknown) signal $S$ from its dendritic system. In this frame, the stochastic Hodgkin-Huxley model is a coupled system of diffusion equations describing the observed membrane potential process (first coordinate) as well as unobserved coordinates which model ion currents. Observing the first coordinate alone leads to a non-Markovian process.

The main interest of modern neurosciences is to understand how neurons respond to external stimuli. Therefore it is important to build statistical procedures aiming at estimating the unknown signal (or some important features of the signal), based on the observation of the membrane potential process.

In our work we establish “periodic ergodicity” of the process, based on a detailed study of the transition densities of the stochastic Hodgkin-Huxley model. The main difficulty comes from the fact that our model is a highly degenerate diffusion with time inhomogeneous coefficients. Moreover we obtain limit theorems for the sequence of successive spike intervals.

On concavity of the expected value of the first exit time of the Cauchy process

Tadeusz Kulczycki, Wrocław University of Technology

Let $X_t$ be the Cauchy process in $\mathbb{R}^2$, $D \subset \mathbb{R}^2$ an open bounded set and $\varphi(x) = E_x^\tau_D$ the expected value of the first exit time of $X_t$ from $D$. We prove that if $D \subset \mathbb{R}^2$ is a convex bounded domain then $\varphi$ is concave on $D$. To show it we study the Hessian matrix of the harmonic extension of $\varphi$. The key idea of the proof is based on a deep result of Hans Lewy concerning determinants of Hessian matrices of harmonic functions.

What happens to a random walk of chained particles when the chain is very long?

Serge Cohen, University of Toulouse

Consider a random walk of a chain of $K+1$ particles at integer sites, where the chaining keeps each particle at distance 1 from its immediate neighbours. In dimension 1, we showed with Boissard, Espinasse and Norris that the effect of chaining is to slow down the walk by a factor of $2/(K+2)$. In this talk I will make some remarks for the cases when $K$ is infinite.
There is no fractional Brownian fields indexed by the cylinde

Nil Venet, University of Toulouse

When one aims to consider random processes indexed by more general spaces than the real line, Fractional Lévy Brownian fields are a natural generalisation of the fractional Brownian motion. Given any metric space \((E,d)\), one can ask a real valued centred Gaussian random field \((X_x)_{x \in E}\) to verify

\[
\mathbb{E} [(X_x - X_y)^2] = d^{2H}(x,y),
\]

where \(H\) is a fixed positive real number. Fractional Lévy Brownian fields often enjoy properties such as local self-similarity of parameter \(H\) and stationary increments with respect to the isometry group of \((E,d)\). Alas, they do not always exist. It is well known that they do for all \(H \in (0,1]\) when the index set is \(\mathbb{R}^d\), while they only exists for \(H \in (0,1/2]\) when the sphere \(S^d\) is taken as index set. We will explain in this talk that there is no fractional Brownian fields indexed by the cylinder \(S^1 \times \mathbb{R}\).

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On uniqueness of solutions to martingale problems

Jan Kallsen, University of Kiel

A key question in stochastic analysis concerns whether a Markov process is uniquely determined in law by its generator or its symbol, or, in a different language, whether a semimartingale is determined by its local characteristics. This issue can be rephrased as a martingale problem and it corresponds to uniqueness of solutions to ordinary differential equations in deterministic analysis. We argue that - in spite of many well-established results - there is still a gap in the literature at least on processes with jumps. Indeed, it would be desirable to dispose of a Picard-Lindelöf result warranting uniqueness under sufficient smoothness of the coefficients. This talk discusses a new result and limitations in this regard.