Sequence implication classes in open-shop scheduling

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Solutions of scheduling problems are called *sequences*. They can be described using comparability graphs. A *comparability graph* is a graph whose edges can be oriented transitively. Every *transitive orientation* of a graph corresponding to a scheduling problem corresponds to a solution to this problem. A common concept for describing comparability graphs is the consideration of a so-called Γ-*relation* on the edge set E. The transitive closure of this relation is an equivalence relation whose equivalence classes are called *implication classes*.

Sequences on the 'efficiency frontier' are of special interest. These sequences are called *irreducible*. For every given choice of processing times the set of all irreducible sequences contains an optimal solution.

For open-shop scheduling (no presettings for job order or machine order) with makespan criterion the notion of comparability graphs can be used to decide in polynomial time whether a sequence A reduces some other sequence B (see Bräsel et al. [1]). However, it is an open problem to decide in polynomial time whether a given sequence A is irreducible.

To tackle this problem Willenius [2] introduced the notion of *sequence implication classes* as a substructure of implication classes. These sequence implication classes will be introduced and their importance outlined.

References

[1] Bräsel, Heidemarie; Harborth, Martin; Tautenhahn, Thomas und Willenius, Per (1999). On the set of solutions of an open shop Problem. *Ann. Oper. Res. (1999)* **92** 241-263.

[2] Willenius, Per (2000). *Irreduzibilitätstheorie bei Shop-Scheduling-Problemen.* Dissertationsschrift. Shaker Verlag, 2000.