On the Number of Different Alternative Solutions Generated by the Penalty Method

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In discrete optimization problems one is usually interested in generating an optimal solution. Nevertheless in many practical cases it is advisable to generate not only an optimal solution but also alternative solutions to be able to react appropriately to unforeseen events. Here a simple approach is to take the 2nd best solution as the alternative one. Typically it turns out that this solution is pretty similar to the best one, which is a disadvantage in practical sense. A better strategy for generating real alternatives is the penalty method. I applied penalty method to graph problems with an objective function of sum type. Sum type means, that the objective function is the sum of the weights of all edges in the solution. Here it works as follows:

- Create an optimal solution.
- Multiply all weights of edges which are in the best solution with factor $(1+\varepsilon)$ for $\varepsilon > 0$.
- Create an optimal solution in the graph with the modified weights. This solution is called ε-alternative.

I studied the problem how many different alternative solutions can be generated for shortest path problems and for matroid problems. Without loss of generality we assume that the graph has n vertices and is complete. It is known that for shortest path problems examples exist with $\Omega(n^2)$ alternative solutions which are optimal for different penalty parameters. I could show, that there are only $O(n^2)$ alternative solutions for shortest path problems on directed acyclic graphs. For matroid problems like minimum spanning trees it could be shown, that there are only $O(n)$ alternative solutions which can be optimal for any penalty parameter.