

# Production Planning in the Final Assembly of an Automotive Plant

*Claas Hemig & Jürgen Zimmermann*

Clausthal University of Technology

Julius-Albert-Straße 2, 38678 Clausthal-Zellerfeld

{claas.hemig,juergen.zimmermann}@tu-clausthal.de

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## 1 Introduction

The automotive market has been a sellers' market in the middle of the last century and has altered to a buyers' market in the 80s and 90s. The demand for automobiles has changed from the purpose of gaining access to personal mobility to an expression of the buyer's individuality.

The turnaround in the reception of automobiles carried out by the consumers has forced the automotive OEMs to broaden the number of variants. So e.g. BMW produces thousands of variants out of nine models with about  $10^{32}$  variants theoretically available (cf. Meyr (2004)). These possibilities can only be provided by installing a flexible manufacturing system, which is surveyed e.g. in Browne et al. (1984) and Sethi and Sethi (1990). We annotate that for the automotive industry two types of flexibility are mainly relevant, namely product flexibility (cf. e.g. Slack (1987)) and volume flexibility (cf. e.g. Khouja (1998), Jack and Raturi (2002), Askar et al. (2007)). Installing an adequate product flexibility and adjustment screws for the volume flexibility with respect to the given demands' forecast is a task of the strategic production planning level (cf. e.g. Henrich (2002)). Our approach is located on the tactical level and utilizes the given flexibility instruments.

The automotive industry has recognized that the tactical planning of its production capacities and staff size by hand and experience is not efficient due to the working time required for such a planning, the great variety of adjustment options, and the various influences on the production systems. Therefore, in cooperation with our industrial partner we develop a software tool to support the production and staff planners in several plants. The resulting possibility of an immediate application to problems of

real-world size and with real-world data enables us to include requirements from the cutting edge of automotive production planning into our research.

After a detailed problem formulation in Chapter 2, we present our solution approach in Chapter 3. To handle the arising complexity for the case of parallel production lines we disaggregate our holistic approach and focus on one of the two subproblems to be solved and close by some conclusions and an outlook on further research.

## 2 Problem Formulation and Model Building

A typical automotive plant mainly consists of a body shop, a paint shop, and a final assembly. In each shop there may exist several parallel production lines that are organized as a continuous flow production system. On the lines miscellaneous types of products, called models, are manufactured. Each line can either be a solitary line (i.e. only one product can be produced) or a product flexible line (i.e. several different products can be produced). Buffers of limited capacities between the shops can store intermediate products to decouple the production in subsequent shops. In this paper we focus on a final assembly with  $L$  parallel production lines where some or all of  $M$  products can be produced by more than one line.

In particular we investigate the interrelation of the production time, the production speed and the needed amount of workforce on the particular production lines to fulfill a given demand for each product in each period of a planning horizon  $T$ . The planning horizon covers a whole life cycle of a product, i.e. three to six years, where a period has a length of a week or a month. Furthermore, we examine the dependencies of the parallel lines regarding the interchange of staff as well as the distribution of workload. The objective of our approach is to find a cost optimal solution taking into account technical and labor restrictions as well as the given flexibility instruments of the underlying shop.

The following sections describe the decisions of our mathematical model that have to be made for each period. Simultaneously, we introduce the most important restrictions and finalize the chapter by presenting an adequate cost function.

### 2.1 Shift Model, Cycle Time, and Staff Demand

The fundamental decisions concerning the objective target in the final assembly are the shift model  $sm_{lt} \in \mathcal{SM}_l$  and the cycle time  $ct_{lt} \in \mathcal{CT}_l$  for each period  $t$  and each line  $l$ , where the sets  $\mathcal{SM}_l$  and  $\mathcal{CT}_l$  contain all eligible shift models and cycle times of line  $l$ , respectively. These two decisions restrict the feasible region of all other decisions significantly.

Next to several cost values, e.g. shift allowances, a selected shift model  $sm_{lt}$  provides a certain amount of production time in period  $t$  on line  $l$  whereas the cycle time  $ct_{lt}$

determines the number of stations that have to be installed at the line. The number of stations is equal to the numbers of workers employed directly at the line which are supplemented by some additional workers, e.g. foremen.

The selection of the shift model and the cycle time on each line is independent of each other and determines the production capacity of each line as well as the amount of required workers. In principle we can choose both of them arbitrarily every period, but a change of the shift model as well as the cycle time requires significant organizational and technical changes. Consequently, a change is allowed to occur only once in a given number of subsequent periods  $SM_l^{min}$  and  $CT_l^{min}$ , respectively. To minimize idle capacity it is possible to cancel single shifts little by little as long as the capacity exceeds the given demand for products. The installed capacity restricts the production of the given demand which is described in the following section.

## 2.2 Demand and Production

Each period  $t$  the final assembly has to produce a certain amount  $D_{mt}$  for each product  $m$  allowing a given relative deviation  $D^V$ . The decision how many units of product  $m$  are produced on line  $l$  in period  $t$  is denoted by variables  $p_{mkt}$  with

$$D_{mt} \cdot (1 - D^V) \leq \sum_{l=1}^L p_{mkt} \leq D_{mt} \cdot (1 + D^V) \quad \forall m = 1, \dots, M \quad \forall t = 0, \dots, T - 1. \quad (1)$$

In other words, there is a minimal demand  $D_{mt} \cdot (1 - D^V)$  that has to be met and an additional one that may be met up to the maximum demand  $D_{mt} \cdot (1 + D^V)$ .

Allowing some deviation from the given demand means to deduce an over- or under-production  $\Delta p_{mkt}$  where these variables contain the differences between production and demand accumulated over all periods before  $t$ .

If two (or more) products are produced on one line, we can establish a preferred mix ratio  $mix_{l,m,ct}^{pref}$  of the products for each cycle time so that all stations of the line are nearly equally loaded (cf. e.g. Leopold (1997)). Obviously, summing up  $mix_{l,m,ct}^{pref}$  over all models  $m$  equals 1 for each line  $l$  and each cycle time  $ct$ . It is allowed to deviate from  $mix_{l,m,ct}^{pref}$  in a tight range, and each product  $m$  must use the capacity of line  $l$  at least at a given minimal proportion  $mix_{l,m,ct}^{min}$ . Vice versa, we can calculate the maximal proportion for all models  $m$  that can be produced on line  $l$  as

$$mix_{l,m,ct}^{max} = 1 - \sum_{\substack{n=1 \\ n \neq m}}^M mix_{l,n,ct}^{min} \quad \forall l = 1, \dots, L, \quad \forall m = 1, \dots, M, \quad \forall ct \in CT_l. \quad (2)$$

If product  $m$  cannot be produced on line  $l$  using cycle time  $ct$ , we set  $mix_{l,m,ct}^{min} = mix_{l,m,ct}^{pref} = mix_{l,m,ct}^{max} = 0$ . Vice versa, if  $l$  is a solitary line for product  $m$ , the three values are set equal to 1.

## 2.3 Staff

The labor requirements at line  $l$  in period  $t$  arising from the choice of  $sm_{lt}$  and  $ct_{lt}$  can be met by permanent and temporary staff  $ps_{lt}$  and  $ts_{lt}$ . Typically, permanent employees can only be dismissed at a few points of time (e.g. at the end of a quarter) while contracts with temporary ones can expire at the end of any period. Permanent employees are skilled superior to their temporary colleagues and are evidently necessary for a permanently high output quality. To avoid frequent dismissals and hirings due to changes of the demand for workforce, permanent workers can be displaced between the production lines, and for each line a so called working hours summary is installed. On the working hours summary of a line the workers accumulate their over- and undertime of each period to provide flexible working hours (cf. e.g. Delsen et al. (2007)) where the values  $whs_{lt}$ —and therefore the installed flexibility—are bounded by in-company-agreements and labor legislation. Due to the tactical planning horizon, we do not examine every single worker and associated summary, but the installed workforce at each line and the average of all workers' summaries currently employed at that line.

## 2.4 Objective Function

Our objective is to minimize a cost function which is given as the sum of the staff, production, and changing costs over all periods. Due to the tactical planning horizon we discount the costs with an interest rate of  $\alpha$ . The staff costs  $C_t^S$  are composed of the basic wages for the temporary as well as for the permanent employees, shift premiums, social surcharges, and costs for organizational modifications deriving from their hiring, dismissal and displacement to other lines. The production costs  $C_t^P$  are the sum of the variable production cost (see Section 2.2) over all lines and products. The changing costs  $C_t^C$  are invoked by changing the shift model or the cycle time on a line and are modeled as an affine-linear function in the number of workers currently employed at the considered line. So the objective function that is to minimize can be written as

$$\sum_{t=0}^{T-1} (C_t^S + C_t^P + C_t^C) \cdot e^{-\alpha t}. \quad (3)$$

Obviously, the cost function (3) is separable and the restrictions concerning the working hours summary are nonlinear. For these reasons we selected a Dynamic Programming approach (cf. e.g. Bellman (1957), Bertsekas (2000)) to solve the outlined problem.

## 3 A Solution Approach using Dynamic Programming

### 3.1 Basic Approach

The main elements of any Dynamic Programming approach are states, decisions, a separable objective function, and transformation functions. In the following, we introduce these elements for the described planning problem.

A state  $z_t$  associated with some period  $t$  is characterized by the accumulated over- or underproduction  $\Delta p_{mt}$  as well as a number of line-specific state variables. These variables are the actually chosen shift models  $sm_{lt}$  and cycle times  $ct_{lt}$ , the number of periods elapsed since their last changes  $s_{lt}^{sm}$  and  $s_{lt}^{ct}$ , respectively, the numbers of permanent and temporary employees  $ps_{lt}$  and  $ts_{lt}$ , respectively, as well as their working hours summaries  $whs_{lt}$ .

A decision  $x_t(z_t)$  is an element of the set of all possible decisions in state  $z_t$ ,  $X_t(z_t)$ , and contains the determination of the shift models  $\widehat{sm}_{lt}$  and the cycle times  $\widehat{ct}_{lt}$ , the numbers of hirings and dismissals of permanent and temporary employees,  $\Delta ps_{lt}$  and  $\Delta ts_{lt}$ , the numbers of displacements of permanent workers,  $ds_{lkt}$ , as well as the decisions about the number of products manufactured on a certain line,  $p_{mlt}$ .

Given some state  $z_t$  and an associated decision  $x_t(z_t)$ , the transformation function  $g_t(x_t(z_t), z_t)$  generates a new state  $z_{t+1}$  of the next period  $t + 1$ . The chosen shift models and cycle times are taken from  $x_t(z_t)$  by  $sm_{l,t+1} = \widehat{sm}_{lt}$  and  $ct_{l,t+1} = \widehat{ct}_{lt}$ , respectively. The number of employees and the accumulated over- or underproduction are modified using the following balance equations

$$ps_{l,t+1} = ps_{lt} + \Delta ps_{lt} + \sum_{k=1}^L (ds_{klt} - ds_{lkt}) \forall l = 1, \dots, L, \quad \forall t = 0, \dots, T - 1, \quad (4)$$

$$ts_{l,t+1} = ts_{lt} + \Delta ts_{lt} \forall l = 1, \dots, L, \quad \forall t = 0, \dots, T - 1, \text{ and} \quad (5)$$

$$\Delta p_{m,t+1} = \Delta p_{mt} + \sum_{l=1}^L p_{mlt} - D_{mt} \forall m = 1, \dots, M, \quad \forall t = 0, \dots, T - 1. \quad (6)$$

The value for  $s_{l,t+1}^{sm}$ —and in analogy for  $s_{l,t+1}^{ct}$ —is set to  $s_{lt}^{sm} + 1$  if the shift model on line  $l$  does not change from period  $t$  to  $t + 1$  and set to 0 otherwise. The possible values for  $s_{lt}^{sm}$  and  $s_{lt}^{ct}$  for all  $t = 0, \dots, T$  have an upper bound of  $SM_l^{min}$  and  $CT_l^{min}$ .

Cost function (3), introduced in Section 2.4, is separable in the periods and, therefore, the Bellman equation for a forward recursion can be written for all  $t = 0, \dots, T - 1$  as

$$F_{t+1}^*(z_{t+1}) = \min_{\substack{x_t(z_t) \in X_t(z_t) \\ z_{t+1} = g_t(x_t(z_t), z_t)}} ((C_t^S + C_t^P + C_t^C) \cdot e^{-\alpha t} + F_t^*(z_t)) \quad (7)$$

Due to the tactical planning horizon and the huge sizes of the feasible decision and state space it is reasonable and necessary to merge similar states to one. To give an example, we consider only states that—*ceteris paribus*—differ in the number of hired permanent workers in discrete steps of 20. Using this idea we can assure the operative usability of the proposed approach for the case of one single production line. In the case of parallel lines this simplification does not suffice to control the increasing complexity. For this reason, we adopt an idea by Schneeweiß (cf. e.g. Schneeweiß (1998)) and disaggregate the approach into two levels, called Top and Bottom Level.

Both the distribution of the workload between the lines as well as the decisions concerning the staff are dependent on the chosen shift models and cycle times. Furthermore, the latter one is dependent on the former, but not the other way round. In other words, we can disaggregate a decision's construction: First, we determine the shift models and cycle times on all lines. Second, we distribute the workload between the lines, and third, we specify the hirings, dismissals, as well as displacements of staff. In the following, we concentrate on the distribution of workload.

## 3.2 Distribution of Workload

The distribution of workload in some period  $t$  is restricted by two essentials: First, the choice of the shift model and the cycle time on every line determines the total capacity as well as the minimal and maximal capacity for each product (see Sections 2.1 and 2.2). Second, the minimal and additional demand for each product is given as input parameters.

Given some feasible combination of shift models and cycle times for each line we assign the minimal fractions of the capacity to the demand of the related products. This assignment is always possible otherwise the corresponding combination of shift models and cycle times would be infeasible. The remaining workload to be distributed is significantly smaller than before the assignment and will be allocated to the remaining capacities by modeling and solving the problem as a classical transportation problem (cf. e.g. Hitchcock (1941)) where each line asks for and each product supplies some portion of the workload. The transportation unit costs are derived from the variable production costs per unit  $c_{m,l,ct}^p$ .

In particular, from each product, e.g. denoted with  $A$ , we generate two suppliers of our transportation problem, namely  $A_{min}$  offering the remaining minimal demand to

be distributed after the considerations made above, and  $A_{add}$  offering the remaining additional demand of  $A$ .

Similarly to the generation of the suppliers, we separate every production line into as many destinations as products can be produced on that line. We restrict ourselves to just two products per line, but the approach can easily be extended to more products. In the following we model the preferred mix ratio of the selected cycle time on this line and its allowed deviation as mentioned in Section 2.2. Figure 1 illustrates the capacity of a line, denoted by 1, on which the products  $A$  and  $B$  can be produced. The dotted line illustrates the preferred mix ratio, the left and right hatched rectangles represent the minimal capacity for  $A$  and  $B$  that is filled with the accordant product's demand.

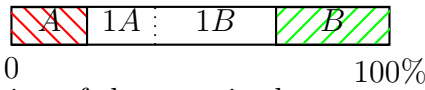


Figure 1: Distribution of the capacity between products A and B

Since the additional demand need not to be produced entirely, we introduce a dummy line to balance the transportation problem and to model unproduced additional demand.

A solution of the proposed transportation problem provides the information whether the remaining, unmarked areas—the yet unused capacity—should be filled with production workload of product  $A$  or  $B$ . Those areas' labels (see Figure 1) indicate the preferred model to be produced there to achieve the preferred mix ratio. The capacity marked with  $1A$  and  $1B$  should be filled with demand of product  $A$  and  $B$ , respectively, so that the preferred mix ratio is met. To achieve this preferred mix ratio we set the transportation costs per unit adequately, i.e. the higher the cost the less is the preference to transport the correspondent supply to the respective destination. If a line cannot produce a product the transportation unit cost is set to a sufficiently large value  $BigM$ .

Specifying the transportation unit costs, we do not differentiate between the two suppliers  $A_{min}$  and  $A_{add}$ , except for the dummy line. The unit cost for transporting from  $A_{min}$  to the dummy line is also set to  $BigM$  because the minimal demand must always be produced on some real lines. We price the transportation from  $A_{add}$  to the dummy line with a cost value greater than all transportation unit cost to any real line, but smaller than  $BigM$ . We establish those transportation costs per unit to introduce a preference order between all products regarding the production of the additional demand, for instance, dependent on the accumulated over- or underproduction.

We solve the transportation problem, add the workload already distributed, and obtain the values for  $p_{mlt}$  for all products  $m$  and all lines  $l$ . The decision still pending is the one concerning the staff. This is described in the following section.

### 3.3 Staff Planning

In order to reduce the computational effort we do not enumerate all feasible decisions concerning the staff. For each partial decision generated as described in Section 3.2 we build up exactly one decision for the staff. This restrictive shortening of considered decisions has to be compensated to avoid infeasibility in further periods. Hence, we incorporate increasing staff demand in the following periods as well as the necessity to take action for a feasible working hours summary.

The determination of the decisions concerning the staff is the last step to complete a decision of the Top Level. We can execute the algorithm of the Top Level as described and obtain an “optimal” solution keeping in mind that we used a heuristic for the decisions concerning the staff.

To improve the solution we start over the Dynamic Programming, fix the determined decision for the shift model, cycle time, and production program and enumerate all meaningful decisions concerning the staff as described in Section 3.1.

Altogether, we complete the decisions adopted from the Top Level by those generated on the Bottom Level and determine an optimal policy keeping in mind the disaggregation.

## 4 Conclusions and Outlook

In this paper we presented a Dynamic Programming approach to solve the problem of simultaneous tactical production and staff planning in the final assembly of an automotive plant. We focused on achieving tractability even for the case of parallel production lines that are not independent of each other and disaggregated the solution approach into two levels. On the Top Level we modeled the problem of distributing the production workload between the production lines as a transportation problem where the planning of the staff size is solved approximately. On the Bottom Level we fixed all decisions beside the staff planning and calculated an optimal staff planning solution.

Areas of future research and development are: First, to integrate the techniques described in this paper into our industrial partner’s software tool which is already able to manage subsequent shops each with one production line and intermediate buffers in between them and second, to improve the Bottom level of the approach. More precisely, the Bottom Level improves the Top Level’s solution just slightly and, therefore, the improvement of the Bottom Level should be done with respect to reduction of time consumption. For instance, a Local Search heuristic, starting with the solution given from the Top Level, might result in such a significant improvement.

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