

Covering of a polygonal region by rectangles

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Abstract

The following 2D covering problem is considered: Let be given a collection of rectangles and a target compact polygonal region. Decide whether there exist translation vectors for the rectangles such that the union of the translated rectangles cover the target polygonal region, or not

In our investigations the covering rectangles can be different from each other, and the target polygonal region can be a nonconvex multi-connected compact object.

We construct a mathematical model of the covering problem based on a special function. This function is a mathematical tool for the analytical description of relationships between the collection of translated covering rectangles and the target polygonal region. In so doing we formalize a cover criterion using the Φ -function technique, i.e. if the target polygonal region is completely contained within the union of translated rectangles then the constructed Φ -function for the region and the complement of the union of translated rectangles has to have a nonnegative value.

This covering problem is multi-extremal and NP-complete. Therefore, we apply a modification of an exact branch-and-bound algorithm. We realize the search for a covering by constructing a search tree and by using of appropriate termination rules. The search ends either with translation vectors for the covering rectangles, i.e. with a covering, or with the result that there does not exist a cover, or with the statement that we can not find a cover.

Computational tests show that the running time substantially depends on the number of rectangles.