

Empty Rectangles in the Unit Square

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At the last WODO 2006 in Siegmundsburg, Dr. G. Scheithauer presented the following problem: On the unit square $[0, 1]^2$ m points are drawn at random, uniformly distributed and independently from each other. For two such points P_i and P_j let R_{ij} be the rectangle spanned by the two points in that way that P_i and P_j are vertices of R_{ij} and sides of R_{ij} are parallel to the sides of the unit square. A rectangle R_{ij} is called *empty* if no other point P_k ($k \neq i, j$) lies inside R_{ij} . An *empty rectangle with minimum size* (p, q) is an empty rectangle with length of at least p and height of at least q . We are interested in the expected number of empty rectangles, both for the unrestricted case and with minimum size.

We give exact formulas for the determination of the expected numbers as well as upper bounds from which we can analyze the asymptotic behaviour. An experimental comparison of exact, asymptotic and simulated values is done.