Schedule execution to minimize makespan for two-machine flow-shop with uncertain processing times

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We address the issue of how to best execute the schedule in a two-phase scheduling decision framework by considering an uncertain two-machine flow-shop scheduling problem $F2|p_{ij}^L \leq p_{ij} \leq p_{ij}^U|C_{max}$ in which each uncertain processing time $p_{ij} \in R_+$ of job $J_i \in \mathcal{J} = \{J_1, J_2, \ldots, J_n\}$ by machine $M_j \in \mathcal{M} = \{M_1, M_2\}$ may take any real value between given lower bound $p_{ij}^L > 0$ and given upper bound $p_{ij}^U \geq p_{ij}^L$. The scheduling objective is to minimize the makespan C_{max}. There are two phases in the scheduling process under consideration: the off-line phase (the schedule planning phase) and the on-line phase (the schedule execution phase). The information of the lower bound p_{ij}^L and upper bound p_{ij}^U for each uncertain processing time p_{ij} is available at the beginning of the off-line phase. The local information on the realization p_{ij}^* (the actual value) of each uncertain processing time p_{ij} is available once the corresponding operation (of a job on a machine) is completed: $p_{ij} = p_{ij}^*$.

Let T denote the set of all possible vectors $p = (p_{1,1}, p_{1,2}, \ldots, p_{n1}, p_{n2})$ of the uncertain processing times: $T = \{p \ | \ p \in R_+^n \ \ p_{ij}^L \leq p_{ij} \leq p_{ij}^U, \ J_i \in \mathcal{J}, \ M_j \in \mathcal{M}\}$. In the off-line phase, a scheduler prepares a minimal set $S(T)$ of dominant permutations [1], which is derived based on a set of sufficient conditions for schedule domination that have been proven in [2, 3]. This set $S(T)$ of dominant permutations enables a scheduler to quickly make an on-line scheduling decision whenever additional local information on a realization of an uncertain processing time is available. Set $S(T)$ also can optimally cover all feasible realizations of the uncertain processing times in the sense that for any feasible realizations p_{ij}^* of the uncertain processing times p_{ij} there exists at least one permutation in set $S(T)$ which is optimal. Our approach enables a scheduler to best execute a schedule and may end up with executing the schedule optimally in many

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instances according to our extensive computational experiments which were based on randomly generated data up to $n \leq 1000$ jobs. The algorithms for testing the set of sufficient conditions of schedule domination are not only theoretically appealing (i.e., polynomial in the number n of jobs) but also empirically fast as our extensive computational experiments indicate.

We would like to mention that there is another scheduling research line in dealing with uncertain processing times, e.g., one with a decision criterion of minimizing the worst-case regret [4, 5, 6]. Basically, the latter scheduling research line deals with the off-line phase only. In this research line, one aims to seek one schedule that is optimal from a decision criterion (i.e., minimizing the worst-case regret) and no attempts are made to take advantage of the local on-line information to best execute the schedule as the scheduled process goes on. As a new scheme dealing with uncertainty, our two-phase scheduling scheme must be tested on a representative class of uncertain scheduling problems. To this end, two-phase scheduling algorithms were coded in C++ and were tested on a large number of randomly generated problems $F2|p_{ij}^L \leq p_{ij} \leq p_{ij}^U|C_{max}$. The computational results obtained in our experiments seems to be rather promising especially for on-line scheduling phase.

In particular, the off-line scheduling allowed us to find optimal schedules for small numbers of jobs ($n \leq 40$) and small errors of input data. For $n > 40$ there were no such randomly generated instances at all. Fortunately, on-line scheduling allowed us to find optimal schedules (with optimality proofs before schedule execution) for most randomly generated instances with small and medium n, $n \leq 100$, and for many randomly generated instances with large n, $200 \leq n \leq 1000$. The value C_{max} obtained for the actual schedule turns out to be equal to the optimal value C^*_{max} calculated for optimal schedule with the actual job processing times p^*_{ij}. It should be noted that value C^*_{max} can be calculated after completing the last job from set $\mathcal{J}$ when all actual job processing times $p^*_{ij} \in T,\ J_i \in \mathcal{J},\ M_j \in \mathcal{M}$, and all actual job completion times become known. The following computational results obtained in our experiments were even more impressive. The average relative error of the makespan $[(C_{max} - C^*_{max})/C^*_{max}] \cdot 100\%$ obtained for all actual schedules was less than 2.9% for all randomly generated instances with small number of jobs: $n \leq 10$. The average relative error of the makespan obtained for all actual schedules was less than 1.67% for all randomly generated instances with n jobs with $20 \leq n \leq 1000$. These results were obtained since the percentage of the correct decisions made in on-line scheduling phase was rather high. Thus, the sufficient conditions for the existence of a dominant permutation [2, 3] may be very effective for on-line scheduling. In should be also noted that the number of decision-making time-points when the order of conflicting jobs has to be decided was rather high for some instances with $n \geq 50$. However, these decisions made in our algorithms were very fast: there were no randomly generated instance which takes a running time more than 0.05 seconds for a processor with 1200 MHz.

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