A new approach to LIBOR modeling

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Interest rates – Notation

- \( T_1 \leq T_2 \leq \cdots \leq T_N = T \): fixed tenor structure with uniform spacing \( \delta = T_{k+1} - T_k \).
- \( B(t, T_k) \): time-\( t \) price of a zero coupon bond maturing at \( T_k \);
- \( L(t, T_k) \): time-\( t \) forward LIBOR for \([T_k, T_{k+1}]\);
- \( F(t, T_k, T_l) \): time-\( t \) forward price for \( T_k \) and \( T_l \);

Fundamental Relationship

\[
F(t, T_k, T_{k+1}) = \frac{B(t, T_k)}{B(t, T_{k+1})} = 1 + \delta L(t, T_k)
\]

- Forward measure \( P_{T_k} \): Corresponds to using \( B(t, T_k) \) as numeraire.
- Terminal measure \( P_T \): equals \( P_{T_N} \); \( B(t, T) \) is the numeraire.
Economic thought (absence of arbitrage) dictates:

**Axiom 1**

The LIBOR rate should be non-negative, i.e. \( L(t, T_k) \geq 0 \) for all \( t \).

**Axiom 2**

The LIBOR rate process should be a martingale under the corresponding forward measure, i.e. \( L(\cdot, T_k) \in \mathcal{M}(P_{T_{k+1}}) \).

Practical applications require:

**Axiom 3**

Models should be analytically tractable (\( \leadsto \) fast calibration).

Models should have rich structural properties (\( \leadsto \) good calibration).
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LIBOR model: Axioms I

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*Models should have rich structural properties (\( \leadsto \) good calibration).*
What axioms do the existing models satisfy?

- LIBOR models
- Forward Price models
Ansatz: model the LIBOR rate as the exponential of a semimartingale $H$ (with tractable structure under $P_T$).

The change of measure from the terminal to the forward measures is of a complicated functional form; the model structure for simple models (e.g. $H$ a Levy process) is not preserved.
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**Consequences for continuous semimartingales:**

1. Caplets can be priced in closed form or by Fourier methods;
2. Swaptions and multi-LIBOR products cannot be priced with Fourier methods;
3. Monte-Carlo pricing is very time consuming $\rightsquigarrow$ coupled high dimensional SDEs!
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**Consequences for general semimartingales:**

1. Even caplets cannot be priced with Fourier methods!
2. Ditto for Monte-Carlo pricing.
Ansatz: model the forward price as the exponential of a semimartingale $H$ (with tractable structure under $P_T$).

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Consequences:

1. the model structure (e.g. $H$ a Levy process) is essentially preserved – but time-inhomogeneity is usually introduced;
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Negative LIBOR rates can occur!
Towards the Affine LIBOR Model

**Aim:** design a model where the model structure is preserved and LIBOR rates are positive.

**Tool:** Affine processes on $\mathbb{R}^d_{\geq 0}$. 
Model Setup:

- $X = (X_t)_{0 \leq t \leq T}$: A time-homogeneous Markov process
- $X$ takes values in $D = \mathbb{R}_d^\geq 0$
- $X$ is called affine, if its moment generating function satisfies

$$E_x \left[ \exp \langle u, X_t \rangle \right] = \exp \left( \phi_t(u) + \langle \psi_t(u), x \rangle \right)$$

for some functions $\phi_t(u), \psi_t(u)$ taking values in $\mathbb{R}$ and $\mathbb{R}^d$ respectively.
\( \phi_t(u) \) and \( \psi_t(u) \) are defined on \([0, T] \times \mathcal{I}_T\), where

\[ \mathcal{I}_T := \left\{ u \in \mathbb{R}^d : E_x \left[ e^{\langle u, X_T \rangle} \right] < \infty, \text{ for all } x \in D \right\} \]

the ‘domain of finite exponential moments’.

Additional technical assumption: \( 0 \in \mathcal{I}^\circ_T \);

The above definition includes the CIR-process, many Levy subordinators, Non-Gaussian Ornstein-Uhlenbeck processes, and multivariate extensions of these.
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Additional technical assumption: \( 0 \in \mathcal{I}^o_T \);

The above definition includes the CIR-process, many Levy subordinators, Non-Gaussian Ornstein-Uhlenbeck processes, and multivariate extensions of these.
The process $X$ is also a regular affine process in the sense of Duffie, Filipovic and Schachermayer [2003], and in particular a semi-martingale.

We can show that the derivatives

$$F(u) := \frac{\partial}{\partial t} \bigg|_{t=0^+} \phi_t(u) \quad \text{and} \quad R(u) := \frac{\partial}{\partial t} \bigg|_{t=0^+} \psi_t(u)$$

exist for all $u \in \mathcal{I}_T$ and are continuous in $u$. Moreover, $F$ and $R$ satisfy the Lévy–Khintchine-type equations:

$$F(u) = \langle b, u \rangle + \int_D \left( e^{\langle \xi, u \rangle} - 1 \right) m(d\xi);$$

$$R_i(u) = \langle \beta_i, u \rangle + \left\langle \frac{\alpha_i}{2} u, u \right\rangle + \int_D \left( e^{\langle \xi, u \rangle} - 1 - \langle u, h^i(\xi) \rangle \right) \mu_i(d\xi),$$

where $(b, m, \alpha_i, \beta_i, \mu_i)_{1 \leq i \leq d}$ are admissible parameters.
Lemma (generalized Riccati equations)

The functions $\phi$ and $\psi$ satisfy the generalized Riccati equations:

$$\frac{\partial}{\partial t} \phi_t(u) = F(\psi_t(u)), \quad \phi_0(u) = 0,$$

$$\frac{\partial}{\partial t} \psi_t(u) = R(\psi_t(u)), \quad \psi_0(u) = u,$$

for all $t \in [0, T]$ and $u \in \mathcal{I}_T$. 
Construction of the martingale $M_t^u$

Let $u \in \mathbb{R}_d^{\geq 0} \cap \mathcal{I}_T$, and define

$$M_t^u = E \left[ e^{\langle X_T, u \rangle} \mid \mathcal{F}_t \right], \quad (t \in [0, T]):$$

1. $M_t^u$ is a martingale:

$$E [M_t^u \mid \mathcal{F}_r] = E \left[ E \left[ e^{\langle X_T, u \rangle} \mid \mathcal{F}_t \right] \mid \mathcal{F}_r \right] = E \left[ e^{\langle X_T, u \rangle} \mid \mathcal{F}_r \right] = M_r^u.$$

2. $M_t^u$ is greater than 1.

3. Due to the affine property of $X$, $M_t^u$ is of the tractable form

$$M_t^u = \exp \left( \phi_{T-t}(u) + \langle \psi_{T-t}(u), X_t \rangle \right).$$
Affine LIBOR model: martingales $\geq 1$

**Construction of the martingale $M^u_t$**

Let $u \in \mathbb{R}^d_{\geq 0} \cap \mathcal{I}_T$, and define

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1. **$M^u_t$ is a martingale:**

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$$E\left[ M_t^u \bigg| \mathcal{F}_r \right] = E\left[ E\left[ e^{X_T \cdot u} \bigg| \mathcal{F}_t \right] \bigg| \mathcal{F}_r \right] = E\left[ e^{X_T \cdot u} \bigg| \mathcal{F}_r \right] = M_r^u.$$

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Affine LIBOR model: Ansatz

Discounted bond prices must satisfy:

\[
\frac{B(\cdot, T_k)}{B(\cdot, T_N)} \in \mathcal{M}(P_{T_N}), \quad \text{for all } k \in \{1, \ldots, N - 1\}.
\]

The Affine LIBOR Model

We model quotients of bond prices using the martingales \( M \):

\[
\frac{B(t, T_1)}{B(t, T_N)} = M^u_{t^{u_1}} \\
\vdots \\
\frac{B(t, T_{N-1})}{B(t, T_N)} = M^u_{t^{u_{N-1}}},
\]

with initial conditions: \( \frac{B(0, T_k)}{B(0, T_N)} = M^{u_k}_0 \), for all \( k \in \{1, \ldots, N - 1\} \).
Proposition

Let \( L(0, T_1), \ldots, L(0, T_N) \) be a tenor structure of non-negative initial LIBOR rates; let \( X \) be an affine process on \( \mathbb{R}^d_{\geq 0} \).

1. If \( \gamma_X > \frac{B(0,T_1)}{B(0,T_N)} \), then there exists a decreasing sequence \( u_1 \geq u_2 \geq \cdots \geq u_N = 0 \) in \( \mathcal{I}_T \cap \mathbb{R}^d_{\geq 0} \), such that

\[
M_0^{u_k} = \frac{B(0, T_k)}{B(0, T_N)}, \quad \text{for all } k \in \{1, \ldots, N\}.
\]

In particular, if \( \gamma_X = \infty \), the affine LIBOR model can fit any term structure of non-negative initial LIBOR rates.

2. If \( X \) is one-dimensional, the sequence \( (u_k)_{k \in \{1, \ldots, N\}} \) is unique.

3. If all initial LIBOR rates are positive, the sequence \( (u_k)_{k \in \{1, \ldots, N\}} \) is strictly decreasing.

\(1\gamma_X := \sup_{u \in \mathcal{I}_T \cap \mathbb{R}^d_{\geq 0}} E_x[e^{\langle u, X_T \rangle}]\)
Forward prices have the following exponential-affine form

\[
\frac{B(t, T_k)}{B(t, T_{k+1})} = \frac{B(t, T_k)}{B(t, T_N)} \frac{B(t, T_N)}{B(t, T_{k+1})} = \frac{M_t^{u_k}}{M_t^{u_{k+1}}}
\]

\[
= \exp \left( \phi_{T_N-t}(u_k) - \phi_{T_N-t}(u_{k+1}) + \langle \psi_{T_N-t}(u_k) - \psi_{T_N-t}(u_{k+1}), X_t \rangle \right).
\]

Now, \( \phi_t(\cdot) \) and \( \psi_t(\cdot) \) are order-preserving, i.e.

\[
u \geq v \Rightarrow \phi_t(u) \geq \phi_t(v) \quad \text{and} \quad \psi_t(u) \geq \psi_t(v).
\]

Consequently: positive initial LIBOR rates lead to positive LIBOR rates for all times.
Forward measures are related via:

\[
\frac{dP_{T_k}}{dP_{T_{k+1}}} \bigg|_{\mathcal{F}_t} = \frac{F(t, T_k, T_{k+1})}{F(0, T_k, T_{k+1})} = \frac{B(0, T_{k+1})}{B(0, T_k)} \times \frac{M_t^{u_k}}{M_t^{u_{k+1}}}
\]

or equivalently:

\[
\frac{dP_{T_{k+1}}}{dP_{T_N}} \bigg|_{\mathcal{F}_t} = \frac{B(0, T_N)}{B(0, T_{k+1})} \times \frac{B(t, T_{k+1})}{B(t, T_N)} = \frac{B(0, T_N)}{B(0, T_{k+1})} \times M_t^{u_{k+1}}.
\]

Hence, we can easily see that

\[
\frac{B(\cdot, T_k)}{B(\cdot, T_{k+1})} = \frac{M_t^{u_k}}{M_t^{u_{k+1}}} \in \mathcal{M}(P_{T_{k+1}}) \quad \text{since} \quad M_t^{u_k} \in \mathcal{M}(P_{T_N}).
\]
The moment generating function of $X_t$ under any forward measure is again of exponential-affine form.

$$E_{P_{T_{k+1}}} \left[ e^{\nu X_t} \right] = M_{0}^{u_{k+1}} E_{P_{T_N}} \left[ M_{t}^{u_{k+1}} e^{\nu X_t} \right]$$

$$= \exp \left( \phi_t (\psi_{T_N-t}(u_{k+1}) + \nu) - \phi_t (\psi_{T_N-t}(u_{k+1})) \right)$$

$$+ \left\langle \psi_t (\psi_{T_N-t}(u_{k+1}) + \nu) - \psi_t (\psi_{T_N-t}(u_{k+1})), x \right\rangle,$$

hence $X$ is a time-inhomogeneous affine process under any $P_{T_{k+1}}$. Note also the “Esscher structure” of the measure change $\frac{dP_{T_{k+1}}}{dP_{T_N}}$. 
The moment generating function of $X_t$ under any forward measure is again of exponential-affine form . . .

$$E_{P_{T_{k+1}}} [e^{\nu X_t}] = M_{0}^{u_{k+1}} E_{P_{T_N}} [M_{t}^{u_{k+1}} e^{\nu X_t}]$$

$$= \exp \left( \phi_t (\psi_{T_N - t}(u_{k+1}) + \nu) - \phi_t (\psi_{T_N - t}(u_{k+1})) + \langle \psi_t (\psi_{T_N - t}(u_{k+1}) + \nu) - \psi_t (\psi_{T_N - t}(u_{k+1)), x) \right),$$

hence $X$ is a time-inhomogeneous affine process under any $P_{T_{k+1}}$.

Note also the “Esscher structure” of the measure change $dP_{T_{k+1}}$. Moreover, denote by $\frac{M_{t}^{u_{k}}}{M_{t}^{u_{k+1}}} = e^{A_k + B_k \cdot X_t}$; then

$$E_{P_{T_{k+1}}} [e^{\nu(A_k + B_k \cdot X_t)}] = \frac{B(0, T_N)}{B(0, T_{k+1})} \exp \left( A'_k + \langle B'_k, x \rangle \right), \quad (1)$$

where $A'_k$ and $B'_k$ are explicitly known in terms of $\phi$ and $\psi$. 

**Affine LIBOR model: dynamics under forward measures**
Affine LIBOR model: caplet pricing (Fourier-methods)

We can re-write the payoff of a caplet as follows (here $\mathcal{K} := 1 + \delta K$):

$$
\delta(L(T_k, T_k) - K)^+ = (1 + \delta L(T_k, T_k) - 1 + \delta K)^+
$$

$$
= \left( \frac{M_T^{u_k}}{M_T^{u_k+1}} - \mathcal{K} \right)^+ = \left( e^{A_k + B_k \cdot X_{T_k}} - \mathcal{K} \right)^+.
$$

(2)

Then we can price caplets by Fourier-transform methods:

$$
C(T_k, K) = B(0, T_{k+1}) E_{P_{T_{k+1}}} \left[ \delta(L(T_k, T_k) - K)^+ \right]
$$

$$
= \mathcal{K} B(0, T_{k+1}) \int_{\mathbb{R}} \mathcal{K}^{iv - R} \frac{\Lambda_{A_k + B_k \cdot X_{T_k}} (R - iv)}{(R - iv)(R - 1 - iv)} dv
$$

(3)

where $\Lambda_{A_k + B_k \cdot X_{T_k}}$ is given by (1).

Similar formula for swaptions (1D affine process).
In some cases even closed-form valuation of caplets is possible!

Suppose that...

- The distribution function of $X_t$ is known, and
- belongs to an exponential family of distributions,

then caplets can be priced by closed form.

Here is an example...
The Cox-Ingersoll-Ross (CIR) process is given by

\[ dX_t = -\lambda (X_t - \theta) \, dt + 2\eta \sqrt{X_t} \, dW_t, \quad X_0 = x \in \mathbb{R}_{\geq 0}, \]

where \( \lambda, \theta, \eta \in \mathbb{R}_{\geq 0} \). This is an affine process on \( \mathbb{R}_{\geq 0} \), with

\[ E_x[e^{uX_t}] = \exp \left( \phi_t(u) + x \cdot \psi_t(u) \right), \]

where

\[ \phi_t(u) = -\frac{\lambda \theta}{2\eta} \log \left( 1 - 2\eta b(t)u \right) \quad \text{and} \quad \psi_t(u) = \frac{a(t)u}{1 - 2\eta b(t)u}, \]

with

\[ b(t) = \begin{cases} t, & \text{if } \lambda = 0 \\ \frac{1-e^{-\lambda t}}{\lambda}, & \text{if } \lambda \neq 0 \end{cases}, \quad \text{and} \quad a(t) = e^{-\lambda t}. \]
CIR martingales: closed-form formula I

Definition

A random variable $Y$ has location-scale extended non-central chi-square distribution, $Y \sim \text{LSNC} - \chi^2(\mu, \sigma, \nu, \alpha)$, if

$$\frac{Y - \mu}{\sigma} \sim \text{NC} - \chi^2(\nu, \alpha)$$

We have that

$$X_t \overset{P_{T_N}}{\sim} \text{LSNC} - \chi^2 \left(0, \eta b(t), \frac{\lambda \theta}{\eta}, \frac{x_a(t)}{\eta b(t)} \right),$$

and

$$X_t \overset{P_{T_{k+1}}}{\sim} \text{LSNC} - \chi^2 \left(0, \frac{\eta b(t)}{\zeta(t, T_N)}, \frac{\lambda \theta}{\eta}, \frac{x_a(t)}{\eta b(t) \zeta(t, T_N)} \right),$$

hence

$$\log \left( \frac{B(t, T_k)}{B(t, T_{k+1})} \right) \overset{P_{T_{k+1}}}{\sim} \text{LSNC} - \chi^2 \left(A_k, \frac{B_k \eta b(t)}{\zeta(t, T_N)}, \frac{\lambda \theta}{\eta}, \frac{x_a(t)}{\eta b(t) \zeta(t, T_N)} \right).$$
Then, denoting by $M = \log \left( \frac{B(T_k, T_{k+1})}{B(T_k, T_{k+1})} \right)$ the log-forward rate, we arrive at:

$$C(T_k, K) = B(0, T_{k+1}) E_{P_{T_{k+1}}} \left[ (e^M - K)^+ \right]$$

$$= B(0, T_{k+1}) \left\{ E_{P_{T_{k+1}}} \left[ e^M 1\{M \geq \log K\} \right] - K P_{T_{k+1}} [M \geq \log K] \right\}$$

$$= B(0, T_k) \cdot \chi^2_{\nu, \alpha_1} \left( \frac{\log K - A_k}{\sigma_1} \right) - K^* \cdot \chi^2_{\nu, \alpha_2} \left( \frac{\log K - A_k}{\sigma_2} \right),$$

where $K^* = K \cdot B(0, T_{k+1})$ and $\chi^2_{\nu, \alpha}(x) = 1 - \chi^2_{\nu, \alpha}(x)$, with $\chi^2_{\nu, \alpha}(x)$ the non-central chi-square distribution function, and all the parameters are known explicitly.

**Similar closed-form solution for swaptions!**
Example of an implied volatility surface for the CIR martingales.
\[ dX_t = -\lambda (X_t - \theta)dt + dH_t, \quad X_0 = x \in \mathbb{R}_{\geq 0} \]

Example of an implied volatility surface for the Γ-OU martingales.
Summary and Outlook

1. We have presented a LIBOR model that
   - is very simple, and yet . . .
   - captures all the important features . . .
   - especially positivity and analytical tractability

2. Future work:
   - thorough empirical analysis
   - extensions: multiple currencies, default risk
   - connections to HJM framework and short rate models

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Thank you for your attention!