Automorphism groups of classical amorphic association schemes of Latin type

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An association scheme is said to be **amorphic** if every possible merging of its classes yields an association scheme. We base our investigation on the family of **classical amorphic** association schemes of order p^2, p an odd prime, and automorphism group $H = \left(\mathbb{Z}_p^2\right) \rtimes \mathbb{Z}_p^*$. By definition, such schemes are mergings of the **complete classical affine** association scheme $\mathscr{A}_p$ of order p^2 and rank $p+2$, as introduced in [2] (see also [5]). Notably, there exists a bijection between classical amorphic schemes and partitions of the point set of the projective line $\mathrm{PG}(1,p)$ of cardinality $p+1$. To each such partition π with s classes of respective cardinalities i_j, $1 \leq j \leq s$, there corresponds an amorphic scheme $\mathscr{M}(\pi)$ of rank $s+1$ whose basis graphs have valency $(p-1)i_j$, $1 \leq j \leq s$. Moreover, the automorphism group $Aut(\mathscr{M}(\pi))$ contains $H \rtimes S$ where S the stabilizer of π in the group $PGL(2,p)$. Note that here π is regarded as an ordered partition. As a consequence we obtain a proof of the following nice folklore result:

Proposition 1.1 *Amorphic scheme $\mathscr{M}(\pi)$ is Schurian if and only if S acts transitively on each class of π.*

An amorphic association scheme is said to be of **Latin type** if each class of its corresponding partition has size of at least 3. In other words, each basis graph of the scheme has valency at least $l(p-1)$, $l \geq 3$ and this naturally corresponds to a set of $l-2$ pairwise orthogonal Latin squares.

Extending investigations in [6], the author NK arranged a new round of computer experimentation aimed at classifying, up to isomorphism, all classical amorphic schemes of Latin type for primes $p \in \{5,7,11,13\}$. Schemes were summarily generated, checked to see if Schurian, and their automorphism groups determined. Full results were obtained for $p = 5,7,11$, and partial results for $p = 13$ (due to limited computer memory). The number of considered schemes is indicated below.

prime p	5	7	11	13
number of schemes	1	4	526	3251

More detailed information about these schemes will be presented in our talk, especially with regard to the following curious observation.

Proposition 1.2 *For all considered values of p and ordered partitions π, one has*

$$Aut(\mathscr{M}(\pi)) \cong H \rtimes S$$

In other words, all automorphisms of association schemes of Latin type are of geometric nature.

In our talk, we shall also discuss recent theoretical activity aimed at extending Proposition 1.2 to all primes p. A promising pathway is suggested, namely the amalgamation of two diverse methodologies: classical results on transitive permutation groups of prime-square order on one hand (e.g., see [1, 4, 7]) and symmetries of nets and Desarguesian planes of order p on the other hand (e.g., see the discussion in [3]).

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