A Collection of Procedures for Working with Directed Strongly Regular Graphs in GAP

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We report about a collection of routines written in computer algebra system GAP [2] which allow efficient working with directed strongly regular graphs. Since the main objects of interest in algebraic graph theory are highly symmetric graphs, strongly regular graphs are playing a central role in this area. One of their possible generalization for directed graphs was given by Duval in 1988, see [1]. These objects started to receive more and more attention recently, therefore we developed a package of routines in GAP in order to make easier working with them. Several successful experimentations have been already reported (see [3, 4]), while other computer results are still waiting for theoretical generalizations.

A *directed strongly regular graph* (DSRG) with parameters (n,k,t,λ,μ) is a regular directed graph on n vertices with valency k, such that every vertex is incident with t undirected edges, and the number of paths of length 2 directed from a vertex x to another vertex y is λ, if there is an arc from x to y, and μ otherwise. In particular, a DSRG with $t=k$ is an SRG, and a DSRG with $t=0$ is a doubly regular tournament. The adjacency matrix $A=A(\Gamma)$ of a DSRG with parameters (n,k,t,λ,μ), satisfies $AJ=JA=kJ$ and

$$A^2 = tI + \lambda A + \mu(J-I-A). \tag{1}$$

Dealing with a DSRG always provides a challenge and poses a lot of questions:

- Does it contain subgraphs with nice properties?
- Can we interpret and generalize the idea of its construction?
- What are its connections to other combinatorial structures?

Answering these questions can be made easier with a few routine inspections which can be left as a job for a computer.

IsDSRG: Checks whether a zero-one matrix A corresponds to a DSRG, or not. In the first step it determines the candidates for the parameters t,λ,μ, after that checks equation (1).

AllInducedDSRGs, AllQuotientDSRGs: For a given graph it computes the system of imprimitivity of its group of automorphisms and checks for all block systems, whether there appear DSRGs among the graphs induced by the blocks, or on the quotient graphs with respect to blocks.

DSRGfromColorGraph: Starting from a color graph it checks all the possibilities for creating digraphs as union of colors, up to algebraic automorphisms. It uses the SetOrbit package written by Pech and Reichard, see [5, 6].

WLClosureOfDSRG: It computes the smallest coherent configuration, which contains the given DSRG. It is based on Matan Ziv-Av's procedure for computing WL-closure (Weisfeiler-Leman closure) of a graph [7].

It is needless to mention the importance of the procedure IsDSRG. Using of procedures AllInducedDSRGs and AllQuotientDSRGs resulted in the understanding of some bigger DSRGs. Their computer-free interpretation on the theoretical level lead to the results published in [4], where we report about a construction which creates bigger DSRGs from smaller ones under certain conditions. The procedure DSRGfromColorGraph played the key role in the discovery of DSRGs as union of relations in association schemes. The results are reported in [3].

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