Classification of discrete group actions on Riemann surfaces of higher genera

Ján Karabáš[1,2], Roman Nedela[1]

[1] *New Technologies for Information Society, University of West Bohemia, Pilsen, Czech republic,*
[2] *Department of Computer Sciences, Faculty of Natural Sciences, Matej Bel University, Banská Bystrica, Slovakia,* karabas@savbb.sk

Discrete actions of finite groups on surfaces appears in many situations in numerous branches of mathematics, cryptography, quantum physics, and many other fields of science. In topological graph theory they can be used to derive lists of highly symmetrical (oriented) maps of fixed genus: regular maps, vertex-transitive maps, Cayley maps, or edge-transitive maps. In particular, the classification of actions of cyclic groups is essential for solving enumeration problems of combinatorial objects, i.e. maps, graphs and others.

The classification of groups acting on the sphere is a classical part of crystallography. In case of torus the situation is in principle known, though there are infinitely many group actions. The problem of classification of discrete actions of groups on orientable surfaces of genera $g \geq 2$ is nowadays challenge. Due to Hurwitz bound there are just finitely many finite groups acting on a surface of given genus $g \geq 2$. The solution to the problem turned to be "realisable": the classification can be done with help of computer algebra systems. Published lists of actions (without help of CAS's) go up to genus five [1, 3, 5].

Using MAGMA [2] we already derived the list of actions of discrete groups on surfaces of genus $2 \leq g \leq 21$ [4]. We shall discuss the details of the procedure, further improvements and applications.

References

[1] O. V. Bogopolski, *Classifying the actions of finite groups on orientable surfaces of genus* 4 *[translation of proceedings of the Institute of Mathematics, 30 (Russian), 48–69, Izdat. Ross. Akad. Nauk, Sibirsk. Otdel., Inst. Mat., Novosibirsk, 1996]*, Siberian Adv. Math. **7**, no. 4, 9–38, Siberian Advances in Mathematics (1997).

[2] W. Bosma, J. Cannon, C. Fieker, and A. Steel, *Handbook of Magma functions*, Version 2.22, Sydney, 2016.

[3] S. A. Broughton, *Classifying finite group actions on surfaces of low genus*, J. Pure Appl. Algebra **69**, no. 3, 233–270 (1991).

[4] J. Karabáš, *Actions of finite groups on Riemann surfaces of higher genera*, http://www.savbb.sk/~karabas/finacts.html, 2013.

[5] A. Kuribayashi and H. Kimura, *Automorphism groups of compact Riemann surfaces of genus five*, J. Algebra **134**, no. 1, 80–103 (1990).