

# Enumeration of actions of cyclic groups on compact closed surfaces

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Between discrete actions of groups on surfaces, the actions of cyclic groups play a central role. They appear naturally as coefficients in enumeration formulae for coverings between manifolds, for maps and hypermaps with given number of edges and in many other problems. Therefore, we need to count the number of them up to an equivalence relation given by the conjugacy of the kernels of the natural epimorphisms from the associated orbifold fundamental groups. For surfaces of small genera  $g$ , the size of the cyclic group is bounded by a linear function of  $g$ . For small genera one can solve the problem by the standard procedure enumerating low index subgroups in a group given by a presentation. The respective commands are implemented in MAGMA or in GAP. In the particular case of cyclic groups, Harvey (1966) derived a criterion for an existence of a cyclic action on a surface of genus  $g$  determining an orbifold with a prescribed signature. In a paper with A. Mednykh (2006) we have derived a multivariable multiplicative function determining the number of cyclic actions on a surface of genus  $g$  of a prescribed signature. The function was determined in an additive form.

Later, V. Liskovets derived an equivalent multiplicative expression of the function. This simplifies the computations significantly, and as a result, we are able to classify the cyclic actions for surfaces of genera up to 300. The tables determining the numbers of cyclic actions were done with the help of the software packages MAGMA and MATHEMATICA.