ACA 2017 Session: Computer Algebra in Algebraic Graph Theory

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1 Organizers

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2 Motivation

The planned session will concentrate on the application of computer algebra in the area of Algebraic Graph Theory (AGT). Symmetry of graphs is one of the central concepts in AGT. A natural way to treat symmetry of a graph Γ is to consider the action of the automorphism group $Aut(\Gamma)$ on various ingredients of Γ. Transitivity of the action of $Aut(\Gamma)$ on the arc set of Γ may be approximated by combinatorial regularity: It is required that all arcs have the same values of suitable local invariants.

One more productive way to approximate symmetry of Γ is to consider its adjacency matrix $A(\Gamma)$. Different matrix algebras, generated by $A(\Gamma)$, together with the spectrum of $A(\Gamma)$, are subjects of diverse successful research lines.

In the case where Γ is a connected regular graph of diameter d, its adjacency (Hoffman) algebra, generated by $A(\Gamma)$, has dimension $d+1$. More generally with an arbitrary graph Γ we associate its Weisfeiler-Leman closure $WL(\Gamma)$. This matrix algebra can be efficiently calculated with the aid of a polynomial time algorithm.

On this way there appears a significant concept of a coherent algebra introduced independently by B. Weisfeiler & A.A. Leman (1968) and D.G. Higman (1970). This is a matrix algebra which satisfies a few additional natural axioms. In fact, it is an axiomatic analogue of the significant concept of the centralizer algebra of a permutation group, which goes back to I. Schur. For each permutation group (G, Ω) the basic graphs of 2-orbits of (G, Ω), in the sense of H. Wielandt (1969), correspond to the members of the standard basis of the centralizer algebra V(G, Ω). In the consideration of graphs symmetry groups this bijection allows to switch freely between relational and matrix languages.

In fact, there are many examples of so-called non-Schurian coherent algebras, which do not appear from a suitable permutation group. Classical examples of such non-Schurian objects are the Shrikhande graph on 16 vertices (1959) and Pasechnik's doubly regular tournament on 15 vertices (1992). Recently, M. Ziv-Av (Ben Gurion University, Israel), using a computer, proved that the smallest non-Schurian coherent algebra corresponds to a configuration on 14 points. Together with M. Klin he managed to elaborate a nice computer-free interpretation of this structure (2016).

A particular case of a coherent configuration (CC), the relational reformulation of coherent algebra, is an association scheme (AS). The concept of AS goes back to the classical school of design of statistical experiments, founded by R.C. Bose. The algebraic reformulation of AS, called now Bose-Mesner algebra, was suggested in 1959. Of special interest are metrical AS's; they correspond to distance regular graphs. In modern terms a regular connected graph Γ of diameter d is called a distance regular graph (DRG) if $WL(\Gamma)$ has dimension d+1. A connected strongly regular graph is a DRG of diameter 2. A surge of interest in SRG's around 1968 and later goes back to the Classification of Finite Simple Groups (CFSG), one of the most striking achievements of modern algebra.

In turn, CFSG made it possible to finish significant projects in AGT, such as the classification of rank 3 graphs and, more generally, of primitive

distance transitive graphs.

All those results and links are perfectly described in a number of well-known texts, by A. Brouwer, A. Cohen, A. Neumaier (1989), N. Biggs (1993), Ch. Godsil, G. Royle (2001), A. Brouwer, W. Haemers (2012). See also the recent lecture notes by M. Klin, S. Gyurki (2015).

All investigations in AGT require very serious computations with the considered graphs, their automorphism groups and related matrix algebras. Step by step some existing powerful computer packages were adapted for these purposes, and others were elaborated. Among these packages are Magma, Sage, and especially GAP. The latter is free, its continuous development is currently coordinated at St. Andrews (Scotland); originally it was developed at Aachen (Germany). Usually, in AGT it is used in conjunction with Grape (L.H. Soicher) and nauty (B. McKay). A lovely text by P. Cameron, "Permutation groups" (1999) provides a few patterns of computations with GAP on the edge of AGT with finite permutation groups.

Note also that AGT has a large intersection with Algebraic Combinatorics (in the sense of E.Bannai, T. Ito (1984)), though nowadays Algebraic Combinatorics has a much wider meaning, covering also such areas as formal power series, enumerative combinatorics, commutative algebra, lattice theory, etc., which typically are not attributed to AGT.

On the other hand, while coherent configurations and association schemes are definitely regarded as the core of AGT many other lines of research play a significant role.

According to the research interest of the organizers and their close colleagues the following areas will be touched in the planned session:

- Coherent configurations and association schemes, in particular strongly regular graphs;
- Regular maps and symmetries of surfaces;
- Group actions on graphs;
- Diverse applications of algebra and linear algebra in AGT;
- Links between association schemes and coding theory.

Of course we mainly encourage submissions which consider these areas in conjunction with computer algebra.

3 Aims and scope

We are soliciting for talks which are devoted to concrete examples of applications of GAP, Magma, Sage in AGT. Reports on investigations with the use of COCO (I. Faradžev, M. Klin, 1991) and its modern developments are also welcome.

The following cornerstones in constructive enumeration of concrete kinds of structures in AGT serve as examples of attractive leads for presentations:

- Enumeration of ASs (A. Hanaki and I. Miyamoto);
- Enumeration of Schur rings over groups of small order (M. Ziv-Av, S. Reichard)
- Attempts of enumeration of all equitable partitions in known primitive triangle-free SRGs;
- Discovery of some classes of non-Schurian ASs and PBIBDs over some transitive action of projective linear groups PSL(2,q) (D. Kalmanovich, S. Reichard);
- Enumeration of self-complementary SRGs appearing as mergings of amorphic ASs (N. Kriger).

Recent progress of L. Babai in the solution of the graph isomorphism problem suggest new possibilities for theoretical investigation of complexity and improvements of known algorithms for computations with CCs and finite permutation groups.

Recently, general theoretical achievements in the theory of regular maps (M. Conder, G.A. Jones, R. Nedela, D. Singerman) led to interesting approaches to the construction of all regular maps over a given highly regular graph (G.A. Jones & M. Ziv-Av). Talks on new results in this direction will definitely be accepted. This topic is closely related to investigation of discrete group actions on surfaces. For surfaces of bounded genus such actions can be computed using computer algebra packages > MAGMA or GAP, see the web pages `http://www.savbb.sk/~karabas/science.html`, or `https://www.math.auckland.ac.nz/~conder/`

In a wider context we are also looking forward to proposed talks related to the use of computer algebra packages for the investigation of diverse classes of maps on graphs, as well as of graph colourings, developing ideas proposed by Yu. Matiyasevich, P. Potočnik, R. Nedela and other experts.

Group actions on graphs is one more area in AGT which is expected to be well-represented in this session.

During the last one to two decades significant progress has been reached in the isomorphism problem for Cayley structures, in particular for circulant combinatorial objects. Among definite leaders in this area are the Israeli experts M. Klin and M. Muzychuk. Thus we naturally expect proposals for talks related to the use of computer algebra tools for the investigation of exceptional properties of Cayley combinatorial structures.

Edge-transitive and arc-transitive graphs form a traditional class of structures which are investigated in AGT using CFSG and deep techniques from group theory. Results by Ch. Praeger, D. Marušič, K. Kutnar as well as their collaborators serve as attractive examples. Typically some infinite classes of graphs are considered, joined by common values of some combinatorial invariants. While the entire result is obtained on a purely theoretical level, consideration of a few exceptional members of an infinite series may require certain computational efforts, thus naturally leading to the use of computers. We also welcome contributions in this area of AGT.

Coding theory and ASs were developing in a strong conjunction from the early birth of these lines of AGT. Many classical books in coding theory (F.J. MacWilliams & N.J.A. Sloane; Ph. Delsarte; A. Barg & S. Litsyn; W.G. Huffman & V. Pless) require from the reader some background in design theory or ASs. Classical classes of ASs such as Hamming schemes and Johnson schemes were originally discovered by the creators of coding theory. Nowadays fruitful interplay of these research areas appears on the crossroads with complexity theory and computer algebra experiments. Indeed, Johnson ASs are in the center of L. Babai's approach to the graph isomorphism problem. On the other hand, modern research with storage systems and devices, data analysis, fault tolerance and other links with applied coding theory can not exist without cleverly arranged computational experiments. Thus submissions in this part of AGT are also encouraged.

4 Focus and organization

The fact that ACA'2017 will take place at Jerusalem provides very promising possibilities to attract the attention of researchers in AGT to this event.

Both co-organizers are active collaborators of Prof. M. Klin (BGU, Beer-Sheva); one of them is his former Ph.D. student. This is why we expect that former students of M.K. from Israel (first of all, his research students) will form the central part of participants of the suggested session on AGT.

Also, both Klin and Muzychuk are well-known outside Israel, therefore we can expect that many of their research partners from Europe will use this possibility to visit Israel.

A few world-renowned research schools in AGT are successfully acting at Tel-Aviv University and Hebrew University. Moreover there are many prominent researchers in AGT who are working in other Israeli schools. We are sure that a number of young researchers from all those places as well as their colleagues from other countries will be interested to attend the planned event.

The final programme and schedule of talks will depend on the number of accepted contributions and the spectrum of research lines to be presented.

If necessary, the organizers may arrange an introductory one-hour invited lecture with the goal of introducing the participants into the major parts of AGT, to be presented at the session.

5 Call for contributions

If you are interested in presenting your recent results in this session, please send your title and abstract to Sven Reichard (cc to Roman Nedela) no later than April 30, 2017, early submissions are appreciated, see the ongoing information about the session on the ACA 2017 homepage.

Each presentation, including Q&A, is 30 minutes. Please use the LaTeX template for your submission, see the page ACA 2017 publications for detailed instructions. Your abstract should be one to two pages long, including references. Please send both the LaTeX source and a compiled PDF version.

As an exception, it is possible to negotiate with the organizers preliminary submission of abstracts in a different format. In any case, once accepted, the abstract should be finally adapted to the standard form.