# Claim Number Processes having the Multinomial Property

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**Abstract**

The present paper provides new characterizations of homogeneous Poisson processes and of mixed Poisson processes. These characterizations are given in terms of the multinomial property. This property is of interest since the finite–dimensional distributions of a claim number process having the multinomial property are completely determined by its one–dimensional distributions, and it is also convenient for statistical purposes since it does not involve any parameters.

## 1 Introduction

Homogeneous Poisson processes and mixed Poisson processes can be characterized in various ways; see e. g. Schmidt [1996] for homogeneous Poisson processes and Grandell [1997] for mixed Poisson processes. The present paper provides additional characterizations of each of these classes of claim number processes. These characterizations are based on the multinomial property and the binomial property.

The main results assert that a claim number process is a homogenous Poisson process if and only if it has independent increments and the binomial property (Theorem 3.2), and that it is a mixed Poisson process if and only if it has the multinomial property (Theorem 4.2).

The multinomial property is important since the finite–dimensional distributions of a claim number process having the multinomial property are completely determined by its one–dimensional distributions. It is also of interest with regard to statistical tests since it does not involve any parameters.

## 2 The Multinomial Property

A stochastic process $\{N_t\}_{t\in\mathbf{R}_+}$ on a probability space $(\Omega, \mathcal{F}, P)$ is said to be a *claim number process* (*without explosion*) if there exists an event $\Omega_0 \in \mathcal{F}$ with $P[\Omega_0] = 0$ such that the following properties are satisfied for all $\omega \in \Omega \setminus \Omega_0$:

– $N_0(\omega) = 0$,
– $N_t(\omega) \in \mathbf{N}_0$ for all $t \in (0,\infty)$,
– $N_t(\omega) = \inf_{s\in(t,\infty)} N_s(\omega)$ for all $t \in \mathbf{R}_+$,
– $\sup_{s\in[0,t)} N_s(\omega) \le N_t(\omega) \le \sup_{s\in[0,t)} N_s(\omega) + 1$ for all $t \in \mathbf{R}_+$, and
– $\sup_{t\in\mathbf{R}_+} N_t(\omega) = \infty$.

Note that the definition excludes the possibility of infinitely many claims occurring in a finite time interval. We assume henceforth that $\Omega_0 = \emptyset$.

A claim number process $\{N_t\}_{t\in\mathbf{R}_+}$ has
– the *multinomial property* if the identity

$$P\left[\bigcap_{j=1}^{m}\{N_{t_j}-N_{t_{j-1}}=k_j\}\right] \;=\; \frac{n!}{\prod_{j=1}^{m} k_j!}\prod_{j=1}^{m}\left(\frac{t_j-t_{j-1}}{t_m}\right)^{k_j}\cdot P[\{N_{t_m}=n\}]$$

holds for all $m \in \mathbf{N}$, all $t_0, t_1, \ldots, t_m \in \mathbf{R}_+$ such that $0 = t_0 < t_1 < \cdots < t_m$ and all $k_1, \ldots, k_m, n \in \mathbf{N}_0$ such that $\sum_{j=1}^{m} k_j = n$, and it has
– the *binomial property* if the identity

$$P[\{N_s=k\}\cap\{N_t-N_s=n-k\}] \;=\; \binom{n}{k}\left(\frac{s}{t}\right)^{k}\left(1-\frac{s}{t}\right)^{n-k}\cdot P[\{N_t=n\}]$$

holds for all $s,t \in (0,\infty)$ such that $s < t$ and all $k, n \in \mathbf{N}_0$ such that $k \le n$.

Of course, the idea of the multinomial property is that the conditional distribution of the incremental claim numbers in finitely many adjacent intervals, given the total number of claims in all of these intervals, is the multinomial distribution with probabilities being proportional to the lengths of the intervals. In the definition of the multinomial property (and the binomial property) we have avoided the use of conditional probabilities since it cannot be excluded in advance that the conditioning events have probability zero; see, however, Lemmas 3.1 and 4.1 below.

The multinomial property is important for two reasons: First, if a claim number process has the multinomial property, then its finite–dimensional distributions are completely determined by its one–dimensional distributions. Second, the multinomial property does not involve any parameters and is thus particularly suitable for statistical tests.

In the literature, the multinomial property has been considered only occasionally; an exception is Schmidt [1996] who characterized homogeneous Poisson processes in terms of the multinomial property and showed that also every mixed Poisson process has the multinomial property. Theorem 4.2 below asserts that a claim number process has the multinomial property if and only if it is a mixed Poisson process.

By contrast, the binomial property has already been considered in the thesis of Lundberg [1940] who proved that a regular claim number process has the binomial property and the Markov property if and only if it is a mixed Poisson process. Theorem 3.2 below asserts that a claim number process has the binomial property and independent increments if and only if it is a homogeneous Poisson process.

# 3 Homogeneous Poisson Processes

Let us first characterize homogeneous Poisson processes. We need the following lemma:

**3.1 Lemma.** *Assume that* $\{N_t\}_{t\in\mathbf{R}_+}$ *is a claim number process. If* $\{N_t\}_{t\in\mathbf{R}_+}$ *has the binomial property, then*

$$P[\{N_t = n\}] \;>\; 0$$

*and*

$$P[\{N_t - N_s = n\}] \;>\; 0$$

*holds for all* $s,t \in (0,\infty)$ *such that* $s < t$ *and all* $n \in \mathbf{N}_0$*.*

**Proof.** Assume first that there exists some $m \in \mathbf{N}_0$ such that

$$P[\{N_t = n\}] \;=\; 0$$

holds for all $t \in (0,\infty)$ and all $n \in \mathbf{N}$ such that $m < n$. Then we have $P[\{N_t \leq m\}] = 1$ for all $t \in (0,\infty)$. Since the paths of a claim number process are increasing, we obtain

$$\begin{aligned} P\left[\left\{\sup_{t\in(0,\infty)} N_t \leq m\right\}\right] &= P\left[\bigcap_{t\in(0,\infty)} \{N_t \leq m\}\right] \\ &= \inf_{t\in(0,\infty)} P[\{N_t \leq m\}] \\ &= 1 \end{aligned}$$

This contradicts our assumption since the paths of a claim number process increase to infinity.

Consider now $m \in \mathbf{N}_0$. By the first part of this proof, there exists some $t \in (0,\infty)$ and some $n \in \mathbf{N}$ such that $m < n$ and

$$P[\{N_t = n\}] \;>\; 0$$

By the binomial property, we have

$$\begin{aligned} P[\{N_s = k\}] &\geq P[\{N_s = k\} \cap \{N_t - N_s = n-k\}] \\ &= \binom{n}{k}\left(\frac{s}{t}\right)^k \left(\frac{t-s}{t}\right)^{n-k} P[\{N_t = n\}] \end{aligned}$$

which implies that the inequality

$$P[\{N_s = k\}] \;>\; 0$$

holds for all $s \in (0,t)$ and all $k \in \{0,1,\ldots,n\}$. Furthermore, for every $u \in (t,\infty)$, the identity $\sum_{p=n}^{\infty} P[\{N_u = p\}|\{N_t = n\}] = 1$ yields the existence of some $p \in \mathbf{N}$ such that $n \leq p$ and

$$\begin{aligned} P[\{N_u = p\}] &\geq P[\{N_u = p\} \cap \{N_t = n\}] \\ &= P[\{N_u = p\}|\{N_t = n\}] \cdot P[\{N_t = n\}] \\ &> 0 \end{aligned}$$

Repeating the argument used before with $u$ and $p$ instead of $t$ and $n$, we see that the inequality

$$P[\{N_s = k\}] \;\; > \;\; 0$$

holds for all $s \in (0,\infty)$ and all $k \in \{0,1,\ldots,n\}$.
Since $m \in \mathbf{N}_0$ was arbitrary, we conclude that the inequality

$$P[\{N_s = k\}] \;\; > \;\; 0$$

holds for all $s \in (0,\infty)$ and all $k \in \mathbf{N}_0$.
Finally, using the binomial property again, it is immediately seen that

$$P[\{N_t - N_s = k\}] \;\; > \;\; 0$$

holds for all $s,t \in (0,\infty)$ such that $s < t$ and all $k \in \mathbf{N}_0$. □

A claim number process $\{N_t\}_{t\in\mathbf{R}_+}$

– has *independent increments* if

$$P\left[\bigcap_{j=1}^{m}\{N_{t_j} - N_{t_{j-1}} = k_j\}\right] \;\; = \;\; \prod_{j=1}^{m} P[\{N_{t_j} - N_{t_{j-1}} = k_j\}]$$

holds for all $m \in \mathbf{N}$, all $t_0, t_1, \ldots, t_m \in \mathbf{R}_+$ such that $0 = t_0 < t_1 < \cdots < t_m$ and all $k_1, \ldots, k_m \in \mathbf{N}_0$, and it

– is a *homogeneous Poisson process with parameter* $\alpha \in (0,\infty)$ if

$$P\left[\bigcap_{j=1}^{m}\{N_{t_j} = n_j\}\right] \;\; = \;\; \prod_{j=1}^{m} e^{-\alpha(t_j - t_{j-1})}\,\frac{(\alpha(t_j - t_{j-1}))^{n_j - n_{j-1}}}{(n_j - n_{j-1})!}$$

holds for all $m \in \mathbf{N}$, all $t_0, t_1, \ldots, t_m \in \mathbf{R}_+$ such that $0 = t_0 < t_1 < \cdots < t_m$ and all $n_0, n_1, \ldots, n_m \in \mathbf{N}_0$ such that $0 = n_0 \le n_1 \le \cdots \le n_m$.

It is immediate from the definition that every Poisson process has independent increments.

We have the following result:

**3.2 Theorem.** *Assume that* $\{N_t\}_{t\in\mathbf{R}_+}$ *is a claim number process. Then the following are equivalent:*

(a) $\{N_t\}_{t\in\mathbf{R}_+}$ *is a homogeneous Poisson process.*

(b) $\{N_t\}_{t\in\mathbf{R}_+}$ *has the binomial property and independent increments.*

(c) $\{N_t\}_{t\in\mathbf{R}_+}$ *has the multinomial property and there exists some* $\alpha \in (0,\infty)$ *such that*

$$P[\{N_t = n\}] \;\; = \;\; e^{-\alpha t}\,\frac{(\alpha\, t)^n}{n!}$$

*holds for all* $t \in (0,\infty)$ *and all* $n \in \mathbf{N}_0$.

**Proof.** It is easy to verify that (a) and (c) are equivalent; see Schmidt [1996; Lemma 2.3.1]. It is also easily seen that (a) implies (b).
Assume now that (b) holds and let us prove (a). Since $\{N_t\}_{t\in\mathbf{R}_+}$ has independent increments, it is sufficient to show that there exists some $\alpha \in (0,\infty)$ such that

$$\begin{aligned} P[\{N_t = k\}] &= e^{-\alpha t}\,\frac{(\alpha\, t)^k}{k!} \\ P[\{N_t - N_s = k\}] &= e^{-\alpha(t-s)}\,\frac{(\alpha\,(t-s))^k}{k!} \end{aligned}$$

holds for all $s,t \in (0,\infty)$ such that $s < t$ and all $k \in \mathbf{N}_0$.
Since $\{N_t\}_{t\in\mathbf{R}_+}$ has independent increments and the binomial property, we have

$$P[\{N_s = k\}] \cdot P[\{N_t - N_s = l\}] = \binom{k+l}{k}\left(\frac{s}{t}\right)^k\left(\frac{t-s}{t}\right)^l \cdot P[\{N_t = k+l\}] \qquad (*)$$

for all $s,t \in (0,\infty)$ such that $s < t$ and all $k,l \in \mathbf{N}_0$. This identity will be used repeatedly.
By Lemma 3.1, we have

$$\begin{aligned} P[\{N_t = n\}] &> 0 \\ P[\{N_t - N_s = n\}] &> 0 \end{aligned}$$

for all $n \in \mathbf{N}_0$. Using $(*)$, we obtain

$$\begin{aligned} P[\{N_s = n+1\}] \cdot P[\{N_t - N_s = 0\}] &= \left(\frac{s}{t}\right)^{n+1} \cdot P[\{N_t = n+1\}] \\ P[\{N_s = n\}] \cdot P[\{N_t - N_s = 0\}] &= \left(\frac{s}{t}\right)^{n} \cdot P[\{N_t = n\}] \end{aligned}$$

and hence

$$\frac{n+1}{s}\,\frac{P[\{N_s = n+1\}]}{P[\{N_s = n\}]} = \frac{n+1}{t}\,\frac{P[\{N_t = n+1\}]}{P[\{N_t = n\}]}$$

as well as

$$\begin{aligned} P[\{N_s = n\}] \cdot P[\{N_t - N_s = 1\}] &= (n+1)\left(\frac{s}{t}\right)^{n}\frac{t-s}{t} \cdot P[\{N_t = n+1\}] \\ P[\{N_s = n\}] \cdot P[\{N_t - N_s = 0\}] &= \left(\frac{s}{t}\right)^{n} \cdot P[\{N_t = n\}] \end{aligned}$$

and hence

$$\frac{1}{t-s}\,\frac{P[\{N_t - N_s = 1\}]}{P[\{N_t - N_s = 0\}]} = \frac{n+1}{t}\,\frac{P[\{N_t = n+1\}]}{P[\{N_t = n\}]}$$

This proves that

$$\alpha := \frac{n+1}{t}\,\frac{P[\{N_t = n+1\}]}{P[\{N_t = n\}]}$$

does not depend on either $t$ or $n$. Therefore, we have

$$\frac{P[\{N_t = n{+}1\}]}{P[\{N_t = n\}]} \;=\; \frac{\alpha\, t}{n+1}$$

which implies that $N_t$ has the Poisson distribution with parameter $\alpha t$.
Using $(*)$ again, it now follows by straightforward calculation that $N_t - N_s$ has the Poisson distribution with parameter $\alpha(t{-}s)$. □

# 4 Mixed Poisson Processes

Let us now characterize mixed Poisson processes. The following lemma provides a considerable improvement of Lemma 3.1, but it relies on a rather deep result:

**4.1 Lemma.** *Assume that $\{N_t\}_{t\in\mathbf{R}_+}$ is a claim number process. If $\{N_t\}_{t\in\mathbf{R}_+}$ has the binomial property, then there exists a probability measure $Q : \mathcal{B}(\mathbf{R}) \rightarrow [0,1]$ with $Q[(0,\infty)] = 1$ such that*

$$P[\{N_t = n\}] \;=\; \int_{(0,\infty)} e^{-\alpha t}\, \frac{(\alpha\, t)^n}{n!}\, dQ(\alpha)$$

*holds for all $t \in (0,\infty)$ and all $n \in \mathbf{N}_0$.*

**Proof.** For all $n \in \mathbf{N}_0$, define a map $\Pi_n : \mathbf{R}_+ \rightarrow [0,1]$ by letting

$$\Pi_n(t) \;:=\; P[\{N_t = n\}]$$

For $s \in (0,\infty)$ and $k \in \mathbf{N}_0$, the binomial property yields, for all $t \in (s,\infty)$,

$$\begin{aligned}
\Pi_k(s) &= P[\{N_s = k\}] \\
&= \sum_{n=k}^{\infty} P[\{N_s = k\} \cap \{N_t{-}N_s = n{-}k\}] \\
&= \sum_{n=k}^{\infty} \binom{n}{k} \left(\frac{s}{t}\right)^k \left(\frac{t-s}{t}\right)^{n-k} P[\{N_t = n\}] \\
&= \sum_{n=k}^{\infty} \binom{n}{k} \left(\frac{s}{t}\right)^k \left(\frac{t-s}{t}\right)^{n-k} \Pi_n(t)
\end{aligned}$$

In particular, we have

$$\Pi_0(s) \;=\; \sum_{n=0}^{\infty} \left(\frac{t-s}{t}\right)^n \Pi_n(t)$$

The power series $\Pi_0$ is absolutely convergent on the interval $[0, 2t]$. Differentiation yields

$$\begin{aligned}\Pi_0^{(k)}(s) &= \sum_{n=k}^{\infty} k!\binom{n}{k}\left(\frac{t-s}{t}\right)^{n-k}\left(-\frac{1}{t}\right)^{k}\Pi_n(t)\\ &= k!\left(-\frac{1}{s}\right)^{k}\sum_{n=k}^{\infty}\binom{n}{k}\left(\frac{s}{t}\right)^{k}\left(\frac{t-s}{t}\right)^{n-k}\Pi_n(t)\\ &= k!\left(-\frac{1}{s}\right)^{k}\Pi_k(s)\end{aligned}$$

and hence

$$\Pi_k(s) = (-1)^k\,\frac{s^k}{k!}\,\Pi_0^{(k)}(s)$$

Since the previous identity is independent of $t \in (s,\infty)$, it follows that the inequality

$$(-1)^k\,\Pi_0^{(k)}(s) \geq 0$$

holds for all $s \in (0,\infty)$ (which means that $\Pi_0$ is completely monotone on $(0,\infty)$). Since the paths of a claim number process are increasing and right continuous with $N_0 = 0$, we also have

$$\begin{aligned}\lim_{s\to 0}\Pi_0(s) &= \sup_{s\in(0,\infty)} P[\{N_s = 0\}]\\ &= P\left[\bigcup_{s\in(0,\infty)}\{N_s = 0\}\right]\\ &= P\left[\left\{\inf_{s\in(0,\infty)} N_s = 0\right\}\right]\\ &= P[\{N_0 = 0\}]\\ &= 1\end{aligned}$$

Now the theorem of Bernstein and Widder yields the existence of a probability measure $Q : \mathcal{B}(\mathbf{R}) \to [0,1]$ with $Q[\mathbf{R}_+] = 1$ such that

$$\Pi_0(s) = \int_{\mathbf{R}_+} e^{-\alpha s}\,dQ(\alpha)$$

holds for all $s \in \mathbf{R}_+$; see e. g. Berg, Christensen and Ressel [1984; Corollary 6.14] or Mattner [1993]. We thus have $\Pi_0(s) = M_Q(-s)$, where $M_Q$ denotes the moment generating function of $Q$. Since $M_Q$ is finite on $(-\infty,0]$, it follows that $\Pi_0$ is infinitely often differentiable on $(0,\infty)$ with

$$\Pi_0^{(k)}(s) = \int_{\mathbf{R}_+} (-\alpha)^k\,e^{-\alpha s}\,dQ(\alpha)$$

for all $s \in (0,\infty)$ and $k \in \mathbf{N}$; see Billingsley [1995; Section 21]. This yields

$$\begin{aligned}\Pi_k(s) &= (-1)^k \frac{s^k}{k!} \Pi_0^{(k)}(s) \\ &= (-1)^k \frac{s^k}{k!} \int_{\mathbf{R}_+} (-\alpha)^k e^{-\alpha s} \, dQ(\alpha) \\ &= \int_{\mathbf{R}_+} e^{-\alpha s} \frac{(\alpha s)^k}{k!} \, dQ(\alpha)\end{aligned}$$

for all $s \in (0,\infty)$ and all $k \in \mathbf{N}_0$.
To complete the proof, let us show that $Q[\{0\}] = 0$. Indeed, since the paths of a claim number process increase to infinity, we have

$$\begin{aligned}0 &= P\left[\left\{ \sup_{s\in(0,\infty)} N_s = 0 \right\}\right] \\ &= P\left[ \bigcap_{s\in(0,\infty)} \{N_s = 0\} \right] \\ &= \inf_{s\in(0,\infty)} P[\{N_s = 0\}] \\ &= \inf_{s\in(0,\infty)} \Pi_0(s) \\ &= \inf_{s\in(0,\infty)} \int_{\mathbf{R}_+} e^{-\alpha s} \, dQ(\alpha) \\ &\geq Q[\{0\}]\end{aligned}$$

Therefore, we have

$$\Pi_k(s) = \int_{(0,\infty)} e^{-\alpha s} \frac{(\alpha s)^k}{k!} \, dQ(\alpha)$$

for all $s \in (0,\infty)$ and all $k \in \mathbf{N}_0$. □

The proof of the previous lemma is inspired by Hofmann [1955].

A claim number process $\{N_t\}_{t\in\mathbf{R}_+}$
– has the *Markov property* if

$$\begin{aligned}&P\left[\bigcap_{j=1}^{m+1} \{N_{t_j} = n_j\}\right] \cdot P[\{N_{t_m} = n_m\}] \\ &\quad = P\left[\bigcap_{j=1}^{m} \{N_{t_j} = n_j\}\right] \cdot P[\{N_{t_m} = n_m\} \cap \{N_{t_{m+1}} = n_{m+1}\}]\end{aligned}$$

holds for all $m \in \mathbf{N}$, all $t_1, \ldots, t_m, t_{m+1} \in (0,\infty)$ such that $t_1 < \cdots < t_m < t_{m+1}$ and all $n_1, \ldots, n_m, n_{m+1} \in \mathbf{N}_0$ such that $n_1 \leq \cdots \leq n_m \leq n_{m+1}$, and it

– is a *mixed Poisson process with mixing distribution* $Q : \mathcal{B}(\mathbf{R}) \to [0,1]$ if $Q[(0,\infty)] = 1$ and if

$$P\left[\bigcap_{j=1}^{m}\{N_{t_j} = n_j\}\right] \;=\; \int_{(0,\infty)} \prod_{j=1}^{m} e^{-\alpha(t_j-t_{j-1})}\,\frac{(\alpha(t_j-t_{j-1}))^{n_j-n_{j-1}}}{(n_j-n_{j-1})!}\,dQ(\alpha)$$

holds for all $m \in \mathbf{N}$, all $t_0, t_1, \ldots, t_m \in \mathbf{R}_+$ such that $0 = t_0 < t_1 < \cdots < t_m$ and all $n_0, n_1, \ldots, n_m \in \mathbf{N}_0$ such that $0 = n_0 \le n_1 \le \cdots \le n_m$.

It is easy to see that every mixed Poisson process has the Markov property.

We have the following result:

**4.2 Theorem.** *Assume that* $\{N_t\}_{t\in\mathbf{R}_+}$ *is a claim number process. Then the following are equivalent:*
(a) $\{N_t\}_{t\in\mathbf{R}_+}$ *is a mixed Poisson process.*
(b) $\{N_t\}_{t\in\mathbf{R}_+}$ *has the binomial property and the Markov property.*
(c) $\{N_t\}_{t\in\mathbf{R}_+}$ *has the multinomial property.*

**Proof.** It is easy to verify that (c) and (b) are equivalent and that (a) implies (c). Assume now that (c) holds. Then (a) follows by straightforward calculation from the multinomial property and Lemma 4.1. □

As a corollary of Theorem 4.2, we obtain the following well–known result:

**4.3 Corollary.** *Assume that* $\{N_t\}_{t\in\mathbf{R}_+}$ *is a claim number process. Then the following are equivalent:*
(a) $\{N_t\}_{t\in\mathbf{R}_+}$ *is a mixed Poisson process with independent increments.*
(b) $\{N_t\}_{t\in\mathbf{R}_+}$ *is a homogeneous Poisson process.*

# 5 Remarks

Under the additional assumption that the claim number process is regular in the sense that it has (transition) intensities, the equivalence of (a) and (b) of Theorem 4.2 is due to Lundberg [1940]; see also Grandell [1997; Theorem 6.1]. Theorem 4.2, however, does not require the claim number process has intensities.

Under the additional assumption that the claim number process has finite expectations, Theorem 4.2 could also be used to give another proof of the fact that a claim number process having the binomial property and independent increments as well as finite expectations is a Poisson process: Since independent increments imply the Markov property, it follows from Theorem 4.2 that the claim number process is a mixed Poisson process. Therefore, the claim number process is a mixed Poisson process with independent increments, and this implies that it is a Poisson process; see e. g. Schmidt [1996; Lemma 4.2.5 and Theorem 4.2.6]. Theorem 3.2, however, does not require that the claim number process has finite expectations.

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November 10, 2011