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Oberseminar Analysis

Title: A well-posedness theory for quasilinear SPDEs with initial data

We will address the matter of obtaining a well-posedness theory for the quasilinear parabolic initial value problem with rough forcing given by

$$\begin{aligned}(\partial_2 - a(u)\partial_1^2 + 1)u &= f \qquad \text{on} \quad \mathbb{R}\times\mathbb{R}_+,\\ u &= g \qquad \text{on} \quad \mathbb{R}\times\{0\}\,,\end{aligned}$$

where the coefficients $a \in C^\alpha(\mathbb{R}\times\mathbb{R}_+)$, $g \in C^\alpha(\mathbb{R})$, and $f \in C^{\alpha-2}(\mathbb{R}\times\mathbb{R}_+)$ with $\alpha \in (\frac{2}{3}, 1)$. All functions and distributions are assumed to be periodic in the spatial direction. Notice that classical Schauder theory would suggest that a solution u of the initial value problem should be of class C^α and $\partial_1^2 u$ of class $C^{\alpha-2}$. This presents a problem since the coefficients a are only α-Hölder continuous, which means that the product $a\partial_1^2 u$ is not classically defined (since $2\alpha - 2 < 0$). The first step to treat the initial value problem is, therefore, giving a meaning to this product. This is done in a manner reminiscent of the rough path theory of Lyons and also the work of Hairer; it is directly motivated by the work on the space-time periodic version of our problem by Otto and Weber. In particular, one uses stochastic properties to define the required products for a certain class of "good" functions and then transfers the definitions via a reconstruction lemma onto functions "modelled" after the good functions. This is a work in progress with Felix Otto and Jonas Sauer.