Wave equation with coloured stable noise

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We study the wave equation

$$\begin{cases} \Big(\dfrac{\partial^2}{\partial t^2} - a^2\Delta\Big)U(x,t) = \dot{Z}^H(x), x \in \mathbb{R}^3, t > 0, \\ U(x,0) = 0, \\ \dfrac{\partial U}{\partial t}(x,0) = 0, \end{cases} \tag{1}$$

where the source is a spatial random noise with symmetric α-stable distribution: it is a derivative $\dot{Z}^H(x)$ of a real anisotropic harmonizable fractional stable field Z^H with Hurst index $H \in (1/2, 1)$ and stability index $\alpha \in (1, 2)$. Thus, the random noise is "coloured" in the sense that its increments are not independent. To give a meaning to this equation requires defining a random measure generated by Z^H and an integral with respect to this measure. As far as we know, such questions did not appear in the previous literature.

We consider a candidate solution given by Kirchhoff's formula

$$U(x,t) = \frac{1}{4\pi a}\int_{y:|x-y|<at} \frac{1}{|x-y|} Z^H(dy)$$

and prove that it is a generalized solution.

Further we take a consistent approach to stochastic integration with respect to Z^H, constructing a measure corresponding to Z^H, proving its σ-additivity in probability, defining an integral with respect to that measure, and proving its properties. We also establish regularity of the solution.

The talk is based on joint research with Larysa Pryhara (Taras Shevchenko National University of Kyiv).