CONVERGENCE THEOREMS FOR POSITIVE SEMIGROUPS ON ORDERED BANACH SPACES

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ABSTRACT. The analysis of the long-term behaviour of positive operator semigroups on L^p-spaces or, more generally, on Banach lattices, is a classical topic in infinite dimensional Perron–Frobenius theory and has found applications in various fields such a partial differential equations, stochastic analysis and mathematical biology.

In this talk we discuss convergence theorems for positive semigroups in the more general setting of ordered Banach spaces. We give on overview over different approaches to that topic, ranging from the seminal work of Krein and Rutman in the midth of the 20th century to very recent contributions to the field. We focus on the following three types of results:

(i) Theorems which do not only assume positivity, but even some kind of *strong* positivity (or "positivity-improving property") for the semigroup. This is often sufficient to prove convergence.
(ii) So-called *lower bound theorems*, i.e. results which assume that the orbits of the semigroup asymptotically dominate a fixed vector and which derive convergence from this assumption. Those results originally go back to an idea of Lasota and Yorke.
(iii) Results which combine regularity assumptions on the semigroup with geometric assumptions on the cone of the underlying Banach space. Those results originate in the theory of Banach lattices, but they can be extended far beyond this classical scope.

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