Energy Decay for the linear Damped Klein Gordon Equation on Unbounded Domain.

1 Abstract

In this talk, we consider energy decay for the damped Klein-Gordon equation.

(1.1) $$u_{tt} + \gamma(x)u_t - u_{xx} + u = 0. \ (x,t) \in \mathbb{R} \times \mathbb{R}$$

Where $\gamma(x)u_t$ represents a damping force proportional to the velocity u_t

We give an explicit necessary and sufficient condition on the continuous damping functions $\lambda \geq 0$ for which the energy $E(t) = \int_{-\infty}^{\infty} |u_x|^2 + |u|^2 + |u_t|^2 dx$ decays exponentially, whenever $(u(0), u_t(0)) \in H^2(\mathbb{R}) \times H^1(\mathbb{R})$. The approach we use in this paper is based on the asymptotic theory of C_0 semigroups, in particular the results by Gearhart-Pruss, and later Borichev and Tomilov in which one can relate the decay rate of energy and the resolvent growth of the semigroup generator. A key ingredient of our proof is an projection method, in which we project the frequency domain on appropriate regions and estimate the resolvent norms through Fourier transformation. At the end of talk, I will also show some result on Fractional type Klein Gordon equation.