Problem 1
Recall the definition of the full product $A \mathbin{\hat{\boxtimes}} B$ for relational structures $A$ and $B$ with disjoint relational structures: it always contains the relations $\{(a_1, b_1), (a_2, b_2) \mid a_1 = a_2\}$ and $\{(a_1, b_1), (a_2, b_2) \mid b_1 = b_2\}$. Show that without these relations, $\text{Aut}(A \mathbin{\hat{\boxtimes}} B)$ is in general not isomorphic to $\text{Aut}(A) \mathbin{\hat{\boxtimes}} \text{Aut}(B)$ (give counterexamples!).

Problem 2
The finitary alternating group $A$ on $\mathbb{N}$ is the set of all permutations of $\mathbb{N}$ that can be written as a composition of an even number of transpositions. Show that $A$ is a normal subgroup of $\text{Sym}(\mathbb{N})$. What is the cardinality of $A$?

Problem 3
Let $G$ be a permutation group on a set $X$. Let $F(n)$ be the number of orbits for the componentwise action on $n$-tuples with pairwise distinct entries. Prove that

$$F(n) \leq F(n + 1)$$

Problem 4
Show that a permutation group $G$ on a set $A$ is highly transitive if and only if $\overline{G} = \text{Sym}(A) = \text{Aut}(A;=)$. 